The ^{12}C nucleus as a 3α system with forbidden states

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(Submitted March 14, 1974)
ZhETF Pis. Red. 19, 542–544 (April 20, 1974)

It is shown that the projection in the 3α system, of states forbidden by the Pauli principle in α-α pairs, is a sufficiently strong condition capable of preventing compression of the 3α system.

We consider the specific case of a system of three compound particles, in which each pair is subject to an attraction potential that has bound states forbidden by the Pauli principle. Such models were never considered before.

It was found recently[1] that the interaction of the lightest nuclei is described with good accuracy of attracting nuclear potentials that are practically equivalent to the method of resonant groups,[2] and the scattering phase shifts obey the generalized Levinson theorem[3]

$$\delta_l(0) - \delta_l(\infty) = \pi(a_l + b_l), \tag{1}$$

where a_l and b_l are the numbers of allowed and forbidden bound states with angular momentum l, respectively.

For $\alpha\alpha$ interaction, such a potential takes the form

$$V(r) = V_0/\{1 + \exp[(r - r_0)/a]\} + V_{\text{Coul}}(r) \qquad (V_0 = -125.0\ \text{MeV},\ r_0 = 1.78\ \text{F},\ a = 0.66\ \text{F}). \tag{2}$$

The forbidden deep-lying states in this potential, $0S$, $2S$, and $2D$ [they correspond to eight-nucleon shell configurations $(0s)^8$ and $(0s)^6(1p)^2$ respectively] are approximated with good accuracy by oscillator functions with $\hbar\omega = \hbar\omega\alpha = 22.5$ MeV. For the expansion of the wave function of the 3α system it is natural to use the complete basis of translationally-invariant oscillator shell model (TISM),[4] which has become so popular of late, with the value $\hbar\omega$ given above. To ascertain which states of the 3α system should be discarded from this basis because the two-particle states $0S$, $2S$, and $2D$ are forbidden, we multiplied the oscillator wave functions $|nlm(\mathbf{r}_{12})\rangle$ of the states forbidden with respect to the degree of freedom $\mathbf{r}_{12}$($nl=00$, 20, 22) by the oscillator functions of the second Jacobi coordinate $\rho_3=(\mathbf{r}_1+\mathbf{r}_2)/2-\mathbf{r}_3$, and make up with the aid of the Clebsch-Gordan coefficients for the SU_3 group a linear combination with a definite SU_3 symmetry

$$|n, N-n(\lambda\mu)\omega LM\rangle = \sum_{l\Lambda}\langle n0)l(N-n, 0)\Lambda|(\lambda\mu)\omega L\rangle|nl(\mathbf{r}_{12}); N-n, \Lambda(\rho_3) : LM\rangle. \quad (3)$$

It is obvious that at $n=0$ only $(\lambda\mu)=(N0)$ is permissible, and at $n=2$ we can have $(\lambda\mu)=(N0)$, $(N-2, 1)$ and $(N-4, 2)$, i.e., given the total number of quanta N, there will be two forbidden states with $\mu=0$, one each with $\mu=1$ and 2, while all the states with $\mu\geq 3$ are allowed (these statements are valid for any $N\geq 8$).

The wave functions (3) are already symmetrical with respect to the P_{12} permutation, and therefore the forbidden states with a complete set of quantum numbers of the basis SU_3 are given by

$$|A=3, N[3](\lambda\mu)\omega LM\rangle_{\text{forb}} = C\hat{S}|n, N-n(\lambda\mu)\omega LM\rangle, \quad (4)$$

where C is a normalization and $\hat{S}=1+P_{13}+P_{23}$ is the symmetrizer. Let us now orthogonalize by the usual methods the TISM basis to the forbideen states (4). [The orthogonalization should in fact be carried out only for the states $(\lambda\mu)$ with $\mu=0$, 1, and 2.] As a result, the wave functions of the allowed states ψ satisfy the conditions

$$\langle\psi_{\text{all}}|nlm(\mathbf{r}_{ij})\rangle=0, \qquad ij=12, 13, 23. \quad (5)$$

At $N<8$, all the states are forbidden, since the lowest shell configuration $(0s)^4$ $(1p)^8$ corresponds to $N=8$.

After orthogonalizing the basis to the forbidden state (see Table I), we used it to diagonalize the Hamiltonian of the 3α system with $\alpha\alpha$ interaction (2).

Table I. SU multiplets of the TISM basis, orthogonalized to the forbidden states.

N	8	9	10	11	12
$(\lambda\mu)$	(04)	(33)	(24)(62)	(53)(34)	(12,0) (82) (63) (44) (06)

Table II. Convergence of the calculation results.

N_{max}	8	10	12	14	16	18	24
Dimensionality of basis	1	3	7	12	19	28	68
E_{0^+}, MeV	−13.03	−13.09	−14.89	−14.98	−15.04	−15.05	−15.06

The investigation has shown that the binding energy E_b of the 3α system has good convergence when the basis is expanded, i.e., when N_{max} is increased (Table II). The obtained value E_b (theor)$=15.06$ MeV shows that the 3α system turns out to be somewhat overbound [E_b(exp)$=7.28$ MeV].

Earlier studies[5] with L-dependent potentials, containing repulsion at short distances,[6] gave an underbound 3α system. Calculation with the complete TISM basis (without orthogonalization) yields a binding energy 217.3 MeV, so that the orthogonalization prevents the 3α system from "collapsing."

Of course, the ^{12}C nucleus is not completely a 3α system, and it is of interest to see that even a correct allowance for the Pauli principle relative to the mutual motion in each pair of α particles gives quite reasonable results for its binding energy.

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