Two-particle-two-hole levels in nuclei in the Green's function method

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The microscopic theory of the nucleus makes it possible at present to described sufficiently well the "single-phonon" levels in nuclei, which in microscopic language are interpreted as superpositions of single-particle—single-hole configurations ($1p1h$ configurations). However, relatively pure $1p1h$ levels lie only in the very lowest part of the nuclear excitation spectrum. With increasing excitation energy, the role of the states having a more complicated character increases greatly. This is evidenced, in particular, by the existence of "two-phonon" levels (the triplet 0^+, 2^+, 4^+ in spherical nuclei, quadrupole-octupole, dipole-qudrupole levels, etc)[1)]. In the present paper we obtain, within the framework of the Green's function method, microsocopic equations that describe two-phonon levels in nuclei and their connection with single-phonon levels (we describe here only the idea of the approach). Unlike the direct consideration of the aggregate of $1p1h$ and $2p2h$ configurations,[1] which uses the method of equations of motion, the Green's-function methods make it possible to introduce correctly and to use in the formalism of the theory the well-known fact that the $1p1h$ states are collectivized, which is equivalent to expansion in terms of certain "bare" single-phonon states.

We consider a Fermi system with an even number of particles, in which there is no pairing, and confine ourselves to explicit consideration of $1p1h$ and $2p2h$ configurations. In the excitation of such a system, the following elementary processes are possible: $1p1h \to 1p1h$, $1p1h \to 2p2h$, $2p2h \to 1p1h$, and $2p2h \to 2p2h$ (abbreviated $2\to 2$, $2\to 4$, $4\to 2$, $4\to 4$). For a consistent microscopic description of the corresponding excited states it is necessary to write down a coupled system of equations for the 2-, 3-, and 4-particle causal Green's functions, taken in a definite sequence of the times of the single-particle operators. The 2-particle function (which will be designated K_{22}) is considered in detail in[3]. We introduce the 3-particle Green's function K_{24}

$$K_{24}(11',22';33') = i^3 < T[\psi_{(x_1)}\psi(x_2)\psi(x_3)\psi^+(x_1')\psi^+(x_2')\psi^+(x_3')] >$$

$$t_1, t_1' > t_2, t_2', t_3, t_3',$$

where $1 \equiv \{\mathbf{r}_1, s_1, t_1\}$, and $\psi(x)$ are single-particle operators in the Heisenberg representation. The equations for K_{22} and K_{24} are

$$\begin{aligned} K_{22}(11',22') &= G(12)G(1'2') \\ &+ K_{22}(11',33')U_{22}(33',44')G(42)G(4'2') \\ &+ K_{24}(11',33';44')U_{42}(33',44';55')G(52)G(5'2'), \\ K_{24}(11',22'33') & \\ &= K_{22}(11',44')U_{24}(44',55'66')G(52)G(5'2')G(63)G(6'3') \\ &+ K_{24}(11',44'55')U_{44}(44'55',66'77')G(62)G(6'2')G(73)G(7'3'), \end{aligned} \quad (1)$$

where G is the single-particle Green's function and integration over repeated arguments is assumed. The interaction amplitudes U_{ik} are by definition irreducible in the $1p1h$ and $2p2h$ channels (they do not contain parts that are joined by either $1p1h$ or $2p2h$ lines). For brevity, we do not write out here the terms corresponding to the bound abd nonbound parts of the amplitudes U_{24}, U_{42}, and U_{44}, as well as to all possible permutation of the arguments.

Eqs. (1) describe nuclear states that are superpositions of $1p1h$ and $2p2h$ configurations. It can be shown that under definite assumptions one obtains from Eqs. (1) equations for the single-particle and two-particle density matrices in the generalized random-phase method.[1] The corresponding equations, however, (see[1,3]), are very cumbersome and inconvenient for physical analysis, since the initial equations (1) do not take into account explicitly the collectivization of the $1p1h$ configurations. To do so, we transform Eqs. (1) in such a way that they contain not the $2p2h$ configurations, but certain "2-phonon" configurations. To this end we introduce the Green's function K, which describes the "bare" phonons and satisfies the (symbolic) equation

$$K = GG + KU_{22}GG. \quad (2)$$

The equation for K_{24} can be easily renormalized in the following manner

$$K_{24} = K_{22}U_{24}KK + K_{24}U'_{44}KK, \quad (3)$$

where

$$U'_{44} = U_{44} - K^{-1}U_{22} - U_{22}K^{-1} - U_{22}U_{22}.$$

We now introduce the complete interaction amplitudes Γ_{22} and Γ'_{24}:

$$K_{22} = GG + GG\Gamma_{22}GG, \quad K_{24} = GG\Gamma'_{24}KK \quad (4)$$

we then obtain from (1), (3), and (4) equations for Γ_{22} and Γ'_{24}

$$\begin{aligned} \Gamma_{22} &= U_{22} + \Gamma_{22}GGU_{22} + \Gamma'_{24}KKU_{42}, \\ \Gamma'_{24} &= U_{24} + \Gamma_{22}GG\,U_{24} + \Gamma'_{24}KKU'_{44}. \end{aligned} \quad (5)$$

Let us examine the properties of the function K. We change over to the representation of the single-particle wave functions $\phi_{\lambda_1} \equiv \phi_1$.[2] The function K differed from the complete function K_{22} only in that K does not contain the $2p2h$ configurations. We can therefore write K in the form

$$K(11',22',r_1,r_2,\omega) = i\sum_s \left\{ \frac{\chi_{os}(11',r_1)\chi_{so}(22',r_2)}{\omega - \omega_s + i\gamma} - \frac{\chi_{os}(22',r_2)\chi_{so}(11',r_1)}{\omega + \omega_s - i\gamma} \right\} + K_{reg}, \quad (6)$$

where $\tau_1 = t_1 - t'_1$, $\tau_2 = t_2 - t'_2$, the subscript s labels only the $1p1h$ states, and the unknown part K_{reg} can be regarded as a smooth function of ω. The functions χ_s and ω_s determine the properties of the "bare" phonons and, in accordance with (2), satisfy the equation

$$\chi_s = GGU_{22}\chi_s . \tag{7}$$

Eqs. (5) contain integration with respect to the energy variables, both in the regions close to the Fermi surface, and in remote regions in which $G = G_{\text{reg}}$ [2] and $K = K_{\text{reg}}$. We renormalize Eqs. (6) in such a way that they contain explicitly only integration over the regions that are close to the Fermi surface, and the unknown regular parts of the functions G and K are included in the renormalized interaction amplitudes. To this end, we break up the products GG and KK into two terms, $GG = A + B$ and $KK = A^1 + B^1$, where A and A^1 contain the pole terms, and B and B^1 the regular terms. As a result of the renormalization we obtain

$$\begin{aligned} \Gamma_{22} &= F_{22} + \Gamma_{22} A F_{22} + \Gamma'_{24} A^1 F_{42}, \\ \Gamma'_{24} &= F_{24} + \Gamma_{22} A F_{24} + \Gamma'_{24} A^1 F_{44}, \end{aligned} \tag{8}$$

where the renormalized amplitudes F satisfy equations similar to Eq. (10), in which only regular terms B and B^1 are contained, and F_{ik} is replaced by U_{ik}.

Using the expressions $\Gamma_{22} = g_2 g_2/(\omega - \omega_r)$ and $\Gamma'_{24} = g_2 g_4/(\omega - \omega_r)$ for the values of Γ_{22} and Γ'_{24} near the sought pole ω_r, [2,3] we obtain in place of (8) simpler equations for the blocks g_1 and g_4

$$\begin{aligned} g_2(11') &= \sum_{2,2'} g_2(22') A_{22'} F_{22}(22', 11') + \sum_{s,s'} g_4(ss') A^1_{ss'} F_{42}(ss', 11'), \\ g_4(s_1 s_2) &= \sum_{22'} g_2(22') A_{22'} F_{24}(22', s_1 s_2) + \sum_{ss'} g_4(ss') A^1_{ss'} F_{44}(ss', s_1 s_2), \\ F_{42}(ss', 11') &= \sum_{33', 44'} \chi_s(33') \chi_{s'}(44') F_{42}(33', 44'11'); \\ A_{22'} &= \frac{n_2 - n'_2}{\epsilon_2 - \epsilon'_2 - \omega}, \quad A^1_{ss'} = \frac{2\omega_{ss'}}{\omega^2 - \omega^2_{ss'}}, \quad \omega_{ss'} = \omega_s + \omega_{s'} . \end{aligned} \tag{9}$$

Eqs. (9) have been written
when only the bound terms of the amplitudes F_{24}, F_{42}, and F_{44} differ from zero, and it is assumed that the amplitudes F depend little on the energy variables, so that they can be taken outside the integral sign. Use was made of the approximation $\chi_{s0} = \chi_{0s}$, which is valid in the most important case of the effective fields that depend only on the coordinates (for example, "bare" phonons with $I^\pi = 0^+, 1^-, 2^+, 3^-$).

Eqs. (9) yield for the nuclear states eigenfunctions and energy eigenvalues that are linear combinations of $1p1h$ excitations and 2-phonon excitations (which are characterized by the indices s and s'). The effective amplitudes of the quasiparticle interactions F cannot be obtained within the framework of this approach and should be determined from experiment or with the aid of some approximate method.

The obtained equations are convenient in that in place of the cumbersome summation of $2p2h$ configurations, the summation in the $2p2h$ channel is carried out over states s of "bare" phonons, the eigenfunctions and energies of which are determined separately by solving Eqs. (7). This decreases considerably the order of the matrix in the numerical solution and facilitates the qualitative analysis of Eqs. (9), for in the latter case it is possible to use approximately known properties of the single-phonon problem, or to obtain them from experiment. In addition, the use of the Green's function formalism makes it possible to introduce consistently and understand more clearly the meaning of the phenomenological constants that enter in the theory.

[1)] The problem of "extra" levels in nuclei (i.e., levels that do not fit into the $1p1h$ scheme) is apparently also connected with the problem of accounting for configurations more complicated than $1p1h$.

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[1] J. Sawicki, Phys. Rev. 123, 2231 (1962).
[2] A. B. Migdal, Teoriya konechnykh Fermi-sitstem i svoistva yader (Theory of Finite Fermi Systems and Nuclear Properties), Nauka, 1965.
[3] S. P. Kamerdzhiev, Yad. Fiz. 18, 751 (1973) [Sov. J. Nuc. Phys. 18, No. 4 (1974)].