The Hilbert-Schmidt method in the three-particle problem, and determination of the $t \to d + n$ and $t \to d^* + n$ vertex functions

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By solving three-particle integral equations we calculated the vertex constants for the virtual decays $t \to d + n$ and $t \to d^* + n$, where d^* is a singlet deuteron. The values $G^2_{tdn} = 1.92$ F and $G^2_{td^*n} = -0.380$ F are obtained in a model with a spin-dependent separable potential. This is the first time that the constant $G^2_{td^*n}$ has been calculated.

As is well known,[1] three-particle amplitudes admit of a simple resonant expansion, which is obtained by applying a resolvent expansion, i.e., the Hilbert-Schmidt method, to integral equations of the Faddeev type. The Hilbert-Schmidt expansion provides a convenient basis for the understanding of three-nucleon physics; in particular, recent studies have demonstrated the usefulness of this approach for the study of the unitarity condition in a three-nucleon system[2] and for solving integral equations for four nucleons.[3] In the present article we show how the Hilbert-Schmidt method can be used to calculate the vertex constants G of the decays $t \to d + n$ and $t \to d^* + n$ (where d^* denotes a singlet deuteron). We note that the constant G_{td^*n} is a very important quantity that can be used in the analysis of scattering and reactions in nuclear physics, but until recently there were practically no theoretical methods of calculating this constant.

We use in the calculations the same potential model as in[1], namely a spin-dependent separable pair interaction of the Yamaguchi type. In this model, the tritium binding energy is $E_t = 11.03$ MeV.

The vertex constant G_{tdn} corresponding to the virtual decay $t \to d + n$ is determined as the residue of the S-wave doublet amplitude $T(E)$ of the nd scattering, taken at the pole $E = E_t$:

$$G^2_{tdn} = \lim_{E \to E_t} (E - E_t) T(E), \tag{1}$$

where E is the energy of three nucleons in the c.m.s. The normalization of the amplitude is as follows: $T(E) = (-3\pi/m) e^{i\delta} \sin\delta/k$, m is the nucleon mass, $k = \sqrt{(4/3)m(E + E_d)}$, $E_d = -2.23$ MeV is the deuteron binding energy, δ is the scattering phase shift, and $\hbar = c = 1$. $G_{tdn}{}^2$ has the dimension of length and will be expressed in Fermi units. We note that the constant G_{tdn} can be expressed in terms of the coefficient N in the asymptotic wave function of tritium in the $d + n$ channel:

$$\Psi_t(r, \rho) \underset{\rho \to \infty}{\to} N \phi_d(r) (e^{-\kappa\rho}/\rho)\, \xi, \qquad G^2_{tdn} = 27\pi^2 N^2/m^2, \tag{2}$$

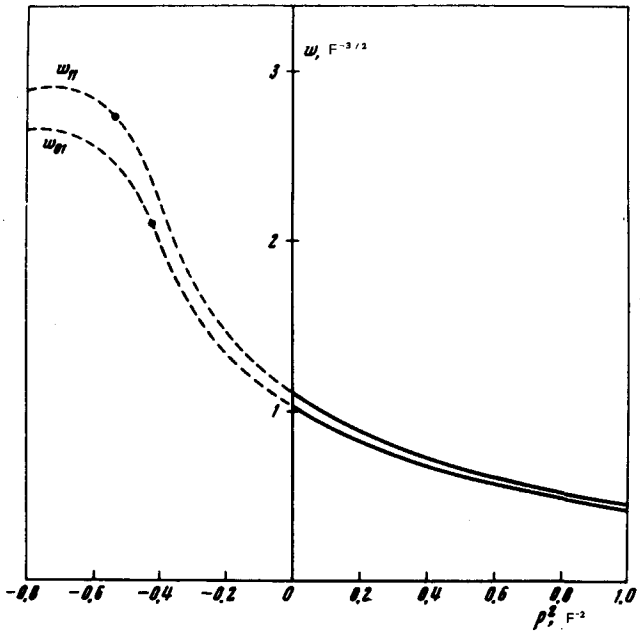
where r is the distance between nucleons in the deuteron, ρ is the distance between the mass center of the deuteron and the third nucleon, $\phi_d(r)$ is the deuteron wave function ($\int |\phi_d(r)|^2 d^3r = 1$), ξ is the spin-isospin function, and $\kappa^2 = (4/3)m(E_d - E_t)$.

We note also that $G_{tdn}{}^2$ is connected with the quantity D^2 used in[4] by the relation $G_{tdn}{}^2 = (3/2)D^2$; since the values of D^2 are expressed in[4] in units of $[10^4 \text{MeV}^2\text{F}^3]$, we have numerically $G^2[\text{F}] = 0.386 D^2$ $[10^4\ \text{MeV}^2\text{F}^3]$.

We use next the resonant expansion obtained in[1] for the amplitude $T(E)$:

$$T(E) = -2(3/2)^{3/2}\pi^2 \frac{\alpha_0(\alpha_0 + \beta_0)}{m} \sum_n w_{0n}(p; E)\theta_n(E) w_{0n}(p; E), \qquad p = \sqrt{\frac{3}{2}}k, \tag{3}$$

where $\alpha_0 = \sqrt{-mE_d} = 0.232$ F^{-1}, $\beta_0 = 1.450$ F^{-1} is the reciprocal radius of the Yamaguchi triplet potential, w_{0n} are the vertices of the transition $\theta_n \rightleftharpoons d + n$ (θ_n is the three-particle Hilbert-Schmidt resonance), and $\theta_n(E)$



The vertex functions $w_{01}(p;E_t)$ and $w_{11}(p;E_t)$. The points on the curves correspond to the values $w_{01}(i\sqrt{3/2}\,\kappa;E_t)$ and $w_1(i\sqrt{3/2}\kappa^*;E_t)$ used to calculate G_{td}^2 and $G_{td^*n}^2$.

$=\eta_n(E)/[1-\eta_n(E)]$ is the θ_n-resonance propagator. The functions $w_{0n}(p;E)$ and $\eta_n(E)$ are defined by expressions (18) and (19) of[1]. As $E\to E_t$ a pole is possessed by only the first term of formula (3); as a result we get for G_{tdn}^2

$$G_{tdn}^2 = 2(3/2)^{3/2}\pi^2\frac{\alpha_0(\alpha_0+\beta_0)}{m\gamma_1}\left[w_{01}\left(i\sqrt{\frac{3}{2}}\kappa;\ E_t\right)\right]^2, \qquad (4)$$

where $\gamma_1=(d\eta_1/dE)_{E=E_t}=0.0350\ \text{MeV}^{-1}$.

The values of the vertex $w_{01}(p;E_t)$ at imaginary p can be calculated with the aid of relation (18) of[1], using the values of the vertices w_{01} and w_{11} (w_{11} is the vertex of the transition $\theta_1\rightleftharpoons d^*+n$) for positive p, which were obtained earlier in the solution of the eigenvalue problem.[2)] The functions $w_{01}(p;E_t)$ and $w_{11}(p;E_t)$ calculated in this manner are shown in the figure. At the point $p=i\sqrt{3/2}\,\kappa$ ($\kappa=0.532\ \text{F}^{-1}$) we have $w_{01}(i\sqrt{3/2}\kappa;E_t)=2.11\ \text{F}^{3/2}$, whence $G_{tdn}^2=1.92$ F. We note that in our method we do not use an analytic approximation of the wave functions.

Calculations previously performed with the aid of the integral-equation method yielded for G_{tdn}^2 the values of 1.0 F (the Reid potential[5]), 1.9 F (the Malflier-Tjon potential[6]), and 0.1 F (the Darevich-Green potential[6]). According to[4,7], the value of G_{tdn}^2 for the Yamaguchi potential is 1.4—1.6 F and differs from that calculated in the present paper. Actually, however, these values were obtained not from the numerical solution obtained in[8] for the Faddeev equations, but from an approximate expression for the wave function of tritium, the analytic structure of which corresponds to the use of a separable potential, and all but one of the parameters of which are fitted to the experimental characteristics of ^{3}H.[9] We note in this connection that it follows from[9] that $G_{td^*n}^2\neq 0(\approx 0.01$ F) for the case when the singlet deutron d^* is regarded as a bound $^1S_0$ state of np with zero binding energy, whereas it is easy to see that in such a determination of d^* one should have $G_{td^*n}^2=0$.

The correct expression for $G_{td^*n}^2$ can be obtained by taking the residue in the total three-particle amplitude at the pole with respect to the variable paired energy of the np system on the second sheet. As a result, we obtain again expression (4), in which we must make the substitutions $\alpha_0\to\alpha_1=-0.0415\ \text{F}^{-1}$, $\beta_0\to\beta_1=1.165\ \text{F}^{-1}$, and $w_{01}(i\sqrt{3/2}\kappa;E_t)\to w_{11}(i\sqrt{3/2}\kappa^*;E_t)$, where α_1 is the wave number of the singlet deuteron, β_1 is the reciprocal radius of the singlet Yamaguchi potential, and $\kappa^*=\sqrt{4m(-\alpha_1^2/m-E_t)/3}=0.594\ \text{F}^{-1}$. Using the value $w_{11}(i\sqrt{3/2}\kappa^*;E_t)=2.72\ \text{F}^{3/2}$, we obtain $G_{td^*n}^2=-0.380$ F. We can also introduce a modified constant $\tilde{G}_{td^*n}^2=mG_{td^*n}^2/2\alpha_1$, which differs from zero also when $\alpha_1\to 0$. The quantities $\tilde{G}^2$ appear in natural fashion, for example, when the reactions (t,N) are described within the framework of the two-nucleon transfer mechanism, or if we use the zero-radius or effective-radius approximation instead of the pole approximation for the interaction amplitude of the transferred pair of nucleons. We then obtain for $\tilde{G}_{td^*n}^2$ the value 21.8 F; for comparison we indicate that $\tilde{G}_{tdn}^2=(m/2\alpha_0)G_{tdn}^2=19.7$ F.

We emphasize that the quantity $G_{td^*n}^2$ is connected by a relation of the type (1) with the amplitude of the nd^* scattering in the state $^2S_{1/2}$, $T=1/2$. If we substitute in (1) instead of an amplitude with a definite total isospin an amplitude with definite projections of the scattered particles, then the value of $G_{td^*n}^2$ and $\tilde{G}_{td^*n}^2$ given above should be multiplied by $\langle 10^{1/2}-1/2;1/2-1/2\rangle^2=1/3$. For the $t\to d+n$ vertex, these two definitions coincide, since the corresponding isospin factor is equal to unity.

The value $G_{tdn}^2=1.92$ F obtained in the present paper exceeds the phenomenological values of G_{tdn}^2 obtained from an analysis of the reactions (t,d) and (d,t) and from the reactions with the lightest nuclei within the framework of the peripherial model[10] ($G_{tdn}^2=1.11\pm 0.05$ F) and the distorted-wave method[4] (1.04 ± 0.19 F), and also from the dispersion relations for the amplitude of forward elastic nd scattering[11] (1.00 ± 0.12 F). This circumstance is not surprising, since the potential used by us is not realistic and is chosen only for the sake of simplicity in the illustration of the method. We note in this connection that if we use in formula (4) not $\kappa=0.532\ \text{F}^{-1}$, which corresponds to $E_t=-11.03$ MeV, but $\kappa=0.449\ \text{F}^{-1}$, which corresponds to the experimental value $E_t=-8.48$ MeV, then we obtain $G_{tdn}^2=1.10$ F, but this procedure is hardly legitimate.

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