Galvanomagnetic properties of metals with closed Fermi surfaces

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We investigate the transverse electric conductivity of pure metals with closed Fermi surfaces in strong magnetic fields at low temperatures.

If the Fermi surface is closed and the number of electrons is not equal to the number of holes, then at temperatures $T \ll T_0 \approx \Delta p s$ the probability of the Umklapp process in electron-phonon collisions, together with the resistivity of an ideal metallic sample, is proportional to $\exp(-T_0/T)$. (Here Δp is the minimal distance between the isolated electron or hole groups, and s is the speed of sound). There are still no reliable data indicating experimental observation of the Peierls exponential, although for certain metals the measurements were performed at temperatures considerably below T_0.

In a preceding paper[1] we have shown, on the basis of a detailed analysis of the mechanism of electron-phonon resistance, that the exponential dependence should become manifest in pure form at temperatures much lower than T_0. Physically this is connected with the fact that at low temperatures Umklapp processes are possible only in small regions (lunes) on the Fermi surface. The electric conductivity σ is then proportional to the total relaxation time within which the electron executes a closed cycle in p-space, namely diffusion through the Fermi surface and a jump between the equivalent lunes as a result of the Umklapp, $\sigma \sim \tau_d + \tau_u$. The diffusion time is $\tau_d \sim T^{-5}$ and the Umklapp time is $\tau_u \sim \exp(T_0/T)$. However, as shown in[1], at $T = T_0$ we have $\tau_d \gg \tau_u$, and consequently the Peierls exponential appears in $\sigma(T)$ at temperatures $T_u \ll T_0$. (The ratio T_u/T_0 decreases with increasing parameter $p_F/\Delta p$, but even for Na and K, for which $p_F/\Delta p \approx 3$, we have $T_u/T_0 \sim 1/10$). Under real conditions, at such low temperatures the scattering of the electrons occurs on defects of the crystal lattice or on the boundaries of the sample (see, e.g.,[3,4]).

The situation is entirely different, as will be shown now, in strong magnetic fields. In this case the exponential dependence can become manifest at temperatures comparable with T_0.

At low temperatures, when the thermal momentum T/s of the phonons is small in comparison with all the characteristic parameters of the Fermi surface and the center dimensions, it is natural to use the diffusion approximation.[1,2] We have

$$-\frac{1}{v}\frac{\partial \chi}{\partial t} + \operatorname{div}[\hat{D}(\nabla\chi - \mathbf{a}(\Delta\chi))] + \Pi(\chi) = -e\mathbf{E}\mathbf{n}. \qquad (1)$$

Here $\chi(\mathbf{p})(\partial n/\partial\epsilon)$ is the nonequilibrium increment to the electron distribution function, $n = [\exp(\epsilon - \mu/T) + 1]^{-1}$, $\mathrm{v} = \partial\epsilon/\partial\mathbf{p}$, $\mathbf{n} = \mathbf{v}/\mathrm{v}$, t is the time of revolution along the orbit in the magnetic field, the second term in the left-hand side describes the electron diffusion, and the term Π describes the Umklapp processes.

In a strong magnetic field we have: $\chi = \chi^{(0)} + \chi^{(1)} + \ldots$,

$$\chi^{(0)} = \mathbf{u}\mathbf{p} + f(p_z), \quad \mathbf{u} = cH^{-2}[\mathbf{E}\times\mathbf{H}], \qquad (2)$$

$$-\left(\frac{1}{v}\right)\left(\frac{\partial\chi^{(1)}}{\partial t}\right) + \operatorname{div}[\hat{D}(\nabla f - \mathbf{a}(\nabla f)] + \Pi(\chi^{(0)}) = -eH_z n_z, \ldots \qquad (3)$$

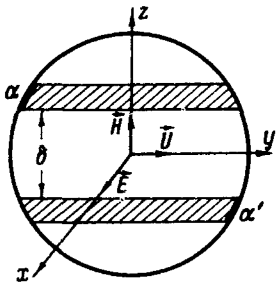
In the calculation of the current, the zeroth approximation (2) is sufficient. However, the function $f(p_z)$ is determined from the conditions for the existence of a solution for Eq. (3):

$$\langle v \operatorname{div}[\hat{D}(\nabla f - \mathbf{a}(\nabla f))] \rangle + \langle v\Pi(\chi^{(0)}) \rangle = -eE_z \langle v_z \rangle, \qquad (4)$$

$$\frac{2}{h^3}\int p_x v\Pi(\chi^{(0)})\,dS = -eE_x(n_e - n_h), \qquad (5)$$

$\langle \ldots \rangle$ denotes the average over the period of revolution.

In the general case of an arbitrary number of lunes and any closed Fermi surface (with a single limitation: the lune dimensions must be small in comparison with the distances between them), the result of solving Eqs. (4) and (5) can be formulated in the form of Kirchoff's laws for the electron currents flowing through branched circuits in momentum space (see[2]). We shall not present here the general expressions for the electric-conductivity tensor. The gist of the matter can be explained from simple physical considerations. In the zeroth approximation in the small parameter $(\Omega\tau)^{-1}$ (Ω is the Larmor frequency and τ is the characteristic relaxation time), only the Hall current $J = ne\underset{\cdot}{u}$ is pro-



duced, and accordingly $\sigma_{xy} = -\sigma_{yx} = necH^{-1}$. The longitudinal electric conductivity σ_{xx} is due to the Umklapp processes. In order of magnitude we have

$$\sigma_{xx} \approx \frac{n_{eff}e^2}{m}\frac{1}{\Omega^2\tau_{eff}}, \qquad (6)$$

where τ_{eff} is the effective free path time relative to collisions with Umklapp, and n_{eff} is the number of electrons that take part in these collisions.

It turns out that the effect of Umklapp processes can be greatly different, depending on whether two or more lunes are contained or are not contained in one section $p_z = \text{const}$. To illustrate this premise, the figure shows a spherical Fermi surface with one pair of equivalent lunes α and α', and the magnetic field is so directed that the lunes do not overlap (the layers of the trajectories passing through the lunes are shown shaded).

At first glance it may appear that owing to the presence of drift (the term $\mathbf{u}\cdot\mathbf{p}$ in (2)) the function χ in the lunes α and α' will be significantly different, and therefore the Umklapp processes will proceed in full force in the magnetic field. In fact, however, if we neglect completely the diffusion, the densities of the nonequilibrium electrons on the equivalent craters under the influence of the Umklapp processes turn out to be equal, and $\sigma_{xx} = 0$. [Formally, the function $f(p_z)$ in (2) is constructed in such a way that the function $\chi^{(0)}$ is the same on the lunes α and α'.] Allowance for the diffusion, of course, leads to a current in the x-axis direction, but in this case we have in (6) $\tau_{eff} \approx \tau_d + \tau_u$, and we arrive at a situation analogous to that considered in[1] (at $H = 0$).

It is now easy to understand that cancellation is impossible if the craters overlap.[1)] Then $\tau_{eff} \approx \tau^U(p_F/r_0)$ and $n_{eff} \approx n(r_0/p_F)$ (r_0 is the radius of the crater and $1/\tau^U$ is the frequency of collisions with Umklapp for an electron situated in the lune). On the other hand, if the craters do not overlap and the distance δ between the layers of the lune trajectories (see the figure) is large enough, so that the number of Umklapp processes is determined by the rate of the electron diffusion (as is possible when $T > T_u$), then $n_{eff} \approx n(\delta/p_F)$ and $\tau_{eff} \approx \tau^{(\delta)}$. Here $\tau^{(\delta)} \approx \tau^F(\delta/p_F)^2$ is the time corresponding to the diffusion displacement over a distance δ, and $\tau^F \sim T^{-5}$ is the usual transport time of electron-phonon interaction.

At an arbitrary field orientation we have, in accorsance with (6),

$$\sigma_{xx} \approx \frac{ne^2}{m}\Omega^{-2}\left[\tau^F\left(\delta/p_F\right) + \tau^U\left(p_F/r_0\right)^2\right]^{-1}. \qquad (7)$$

The Umklapp frequency is

$$1/\tau^U \sim \begin{cases} T^3, & T > T_0 \\ T_0^3 \exp(-T_0/T), & T < T_0. \end{cases}$$

Thus, in strong magnetic fields ($\Omega\tau_{eff} \gg 1$) and at not too low temperatures ($T > T_u$) the electric conductivity turns out to be a strongly anisotropic function of the magnetic field, and for those field directions corresponding to the overlap of the lunes (more accurately, at $\tau^F(\delta/p_F) \ll \tau^U(p_F/r_0)^2$) we have $\sigma_{xx} \sim (\tau^U)^{-1}$. In the latter case of Peierls exponential dependence $\sigma_{xx} \sim \exp(-T_0/T)$ should appear at temperatures $T \sim T_0 \gg T_u$.

[1)]The same result can occur also in the case when nonequivalent craters overlap. It is merely necessary that the electron in the expanded p-space, in the absence of diffusion, to go off to infinity under the influence of the magnetic field and the Umklapp processes.

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