Effect of the character of scattering on the connection between the current and the electric field in metals at low temperatures

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It is shown that inelastic scattering by impurities leads to violation of Ohm's law. The effect can be observed at relatively small current densities in metals if the impurity has a low-energy spectrum.

Deviations from Ohm's law in strong electric fields E in semiconductors have been considered in sufficient detail theoretically and experimentally (see, for example, the review[1]). The effect of strong fields on the characteristics of a degenerate electron gas was considered in[2,3]. The assumptions made in these papers concerning the redistribution of the momentum within the electronic subsystem can be realized at the present time only in semimetals or degenerate semiconductors. However, the experimental results of Yanson and Bogatina[4] show that discernible deviations from Ohm's law are observed in good metals. It will be shown in the present paper than noticeable nonlinearities in relatively weak electric fields can be observed in typical metals in the case of scattering by impurities with interval degrees of freedom at sufficiently low temperatures. It is interesting that the low-energy part of the spectrum of the scattering centers can be reconstructed from the form of these nonlinearities.

In the case when the inequality $\tau_i \ll \tau$ is satisfied (τ_i is the momentum relaxation time[1)] and τ is the effective relaxation time of the energy in the electronic subsystem) one can seek the solution of the kinetic equation in the form of an expansion in the parameter[2)] $\alpha = \sqrt{\tau_i/\tau}$ (see also[5]). The electron distribution function f for an isotropic metal will depend only on ξ and $\cos\theta$ (ξ is the energy reckoned from the Fermi level ϵ_0 and θ is the

angle between the field and the momentum direction). In what follows it will be convenient to divide $f(\xi, \cos\theta)$ into two additive parts, $f_0(\xi)$ and $\Psi(\xi, \cos\theta)$, where f_0 is the Fermi distribution function. Representing $\Psi(\xi, \cos\theta)$ in the form of a series in Legendre polynomials, we retain, accurate to terms of order α^2, only the first two harmonics. Since the elastic collisions make the distribution function isotropic, $\Psi_0(\xi)$ will be of zeroth order in α, and $\Psi_1(\xi)$ of first. The remaining harmonics, generally speaking, will not decrease with increasing number, but the expansion for them will begin with α^2. The equation for Ψ_0 and Ψ_1, accurate to terms $\epsilon/\alpha\epsilon_0$, are

$$\epsilon \frac{\partial \Psi}{\partial \xi} = \frac{\tau_i}{3} I_0\{\Psi_0\}. \tag{1}$$

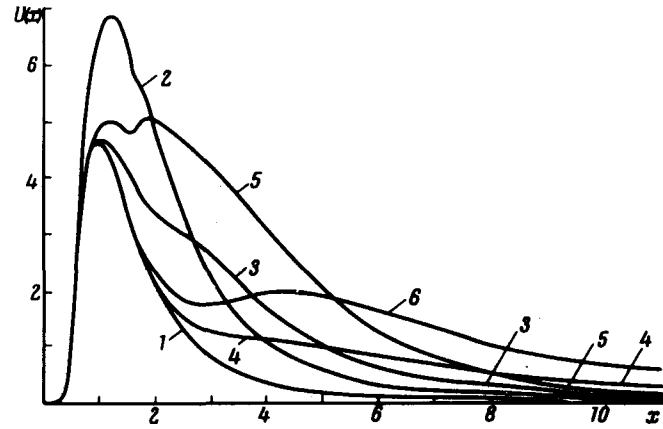
$$\epsilon \left[\frac{\partial \Psi}{\partial \xi} + 2\frac{\partial f_0}{\partial \xi}\right] = -\Psi_1 + \tau_i I_1\{\Psi_0, \Psi_1\}. \tag{2}$$

Here $\epsilon = eEl_i/3$, l_i is the elastic mean free path, and I_0 and I_1 are the zeroth and first harmonics of the inelastic part of the collision integral.

In (1) and (2), the integral I takes into account the collisions with the phonons and the inelastic scattering by the impurities. These equations cannot be solved in the general case, owing to the nonlinear character of the collision term. It is therefore necessary to specify somewhat more concretely the formulation of the problem. We assume first of all the impurity has a discrete energy spectrum, the characteristic energies ω_n of which satisfy the inequality $\omega_n \ll \omega_0$ (ω_0 is the Debye energy). An important role is therefore played only by the energy region $|\xi| \ll \omega_0$. In this region, the phonon part of $I_0\{\Psi_0\}$ can be approximately in the form $-\Psi_0(\xi)/\tau(\xi)$. In a sufficiently general case we have $\tau^{-1}(\xi) \sim |\xi|^\gamma$. Since the physical meaning of $\tau(\xi)$ is the phonon lifetime of the electronic excitation (not the transport lifetime!), it follows that $\gamma = 3$ for a metal, except for a very narrow region $|\xi| \ll \omega_0\sqrt{m/M}$ (m is the electron mass and M is the ion mass), in which $\gamma = 2$.[6] The impurity inelastic part I_0^{in} is equal to zero up to an energy $|\xi| < \omega_1$ (ω_1 is the minimal value of ω_n), and comes into play only at[3] $|\xi| \geq \omega_1$. It is easy to realize experimentally a situation in which $I_0^{in} \ll I_0$ in the energy region $|\xi| \sim \omega_n$. For this purpose it is necessary that the impurities that cause the elastic and inelastic collisions be different, and $c_i \gg c_{in}$ (c_i and c_{in} are the corresponding concentrations), or else, if the impurities are identical, the transitions should be almost forbidden. It must be emphasized that Eqs. (1) and (2) enable us to obtain Ψ_0 accurate to terms $\sim\alpha$, and Ψ_1 accurate to terms $\sim\alpha^2$, since we have discarded in (2) the term with $\epsilon\partial\Psi_2/\partial\xi \sim \alpha^3$. Taking all these remarks into account, we can easily develop a procedure that makes it possible to obtain the inelastic impurity increment Δj to the current $j_0 = \sigma_i E$ (σ_i is the usual conductivity)

$$\Delta j = -j_0 \sum_n \frac{\tau_i}{\tau_n}\left\{2\phi(\omega_n) + \frac{1}{2}\frac{\partial}{\partial\omega_n}\int_{-\infty}^{\infty} d\xi\, \tilde{\Psi}_0(\xi)\tilde{\Psi}_0(\xi+\omega_n)\right\}. \tag{3}$$

Here τ_n is the transport time for inelastic collisions



Dependence of the second derivative of the current with respect to the field on the value of the field: curve 1) $a \to \infty$; 2) $a = 1.2$, $b = 1$; 3) $a = 1.5$, $b = 1$; 4) $a = 2$, $b = 1$; 5) $a = 1.5$, $b = 2$; 6) $a = 2$, $b = 2$; $a = \omega_2/\omega_1$, $b = \tau_1/\tau_2$.

with excitation of impurity from the ground state to the n-th state, $\tilde{\Psi}_0$ ($\xi = \mathrm{sign}\,\xi\,\phi(|\xi|)$ is the zeroth approximation of $\Psi_0(\xi)$ in α,

$$\phi(\omega) = \frac{2}{2^\nu\Gamma(\nu)}\left[\int_0^\omega \frac{d\Omega}{\epsilon}\sqrt{\frac{\tau_i}{3\tau(\Omega)}}\right]^\nu K_\nu\left[\int_0^\omega \frac{d\Omega}{\epsilon}\sqrt{\frac{\tau_i}{3\tau(\Omega)}}\right], \tag{4}$$

$\Gamma(\nu)$ is the Gamma function, K_ν is a Bessel function of imaginary argument, and $\nu = (\gamma+2)^{-1}$.

In the derivation of (3) we have left out the contribution made by the phonon part to I_1, since, first, it contains an additional small factor $\sim(\omega_n/\omega_0)^2$, second, its characteristic scale of field variation is $\epsilon \sim \omega_0$ and not $\epsilon \sim \omega_n\sqrt{\tau_i/3\tau(\omega_n)}$, as in the accounted-for terms. That is to say, under our assumptions the nonlinearity due to this term comes into play in much stronger fields than the nonlinearity from the impurity term. The figure shows the dependence of the derivative of the differential conductivity $\sigma = \partial j/\partial E$ with respect to the field on the value of this field in the relative units

$$u(x) = -\frac{1}{5}\frac{\tau_1\omega_1}{\sqrt{3\tau_i\tau(\omega_1)}}\frac{\partial}{\partial\epsilon}\frac{\sigma}{\sigma_i}, \quad x = \frac{10\epsilon}{\omega_1}\sqrt{\frac{3\tau(\omega_1)}{\tau_i}}$$

for the case when $\gamma = 3$ and the impurity can be in two excited states ω_1 and ω_2. As seen from the figure, the curve reflects the energy structure of the impurity. This structure becomes more clearly pronounced if the transition probability increases with increasing energy. The first maximum of $u(x)$ appears in fields $\epsilon \sim 0.1\omega_1\sqrt{\tau_i/3\tau(\omega_1)}$. If we assume $\omega_1/\omega_0 \sim 10^{-1}$ and $c_i \sim 10^{-2}$, then the current density for a good metal is of the order of 10^4–10^5 A/cm^2, and can be quite easily realized in experiment.[4]

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[1] Transport time.
[2] It should be noted that the expansion takes the form $\Sigma_n \alpha^n A_n(\alpha\xi/\epsilon)$. As will be shown later on, $\alpha\xi/\epsilon$ is not small.
[3] Here and below we assume that $T \ll \omega_1$, so that only the impurity ground state is populated prior to the excitation. In

addition, it is assumed that the phonons give up quite rapidly their excess energy to the thermostat, and the inelastic impurity subsystem also relaxes rapidly via the phonon subsystem.

[4)]A current density 10^9 A/cm^2 was reached in the experiment of Yanson and Bogatina.[4]

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[1]E. Conwell, Kinetic Properties of Semiconductors in Strong Electric Fields (Russ. transl.), Mir. 1970.

[2]Yu. M. Ivanchenko, Zh. Eksp. Teor. Fiz. **59**, 820 (1970) [Sov. Phys.-JETP **32**, 449 (1971)].

[3]L. A. Zil'berman and Yu. M. Ivanchenko, Fiz. Tverd. Tela **14**, 1355 (1972) [Sov. Phys.-Solid State **14**, 1164 (1972)].

[4]I. K. Yanson and N. I. Bogatina, ZhETF Pis. Red. **16**, 395 (1972) [JETP Lett. **16**, 279 (1972)].

[5]B. I. Davydov, Zh. Eksp. Teor. Fiz. 1069 (1937).

[6]A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoi teorii polya v statisticheskoi fizike (Quantum Field Theoretical Methods in Statistical Physics), Fizmatgiz, 1962, p. 245 [Pergamon, 1965].