Excitation of capillary waves in helium by a Wigner lattice of surface electrons

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A description is presented of a resonant method of exciting capillary waves in liquid helium with the aid of surface electrons that form, under certain conditions, a periodic structure on the liquid-vapor interface of liquid helium.

Crandall and Williams[1] And Gor'kov and Chernikova[2] advanced arguments indicating the possibility of ordering of surface electrons on the liquid-vapor boundary of liquid helium into a periodic structure (Wigner crystal). The characteristic dimension r_s of the unit cell of such a structure is estimated at

$$r_s^2 \approx n_s^{-1}, \tag{1}$$

where n_s is the surface density of the electrons.

Using an external clamping electric field $E_\perp^0 \lesssim 10$ cgs esu, the value of n_s can reach $n_s \sim 10^9$–10^{10} cm^{-2}, so that $r_s \approx 10^{-4}$–10^{-5} cm. The "melting" temperature of the surface-electron crystal is estimated[1,3] to be of the order of $T_0 \lesssim 1$ °K under these conditions.

The existence of a periodic lattice of electrons on the free surface of helium in an attainable region of external parameters (temperature and clamping field E^0) can be used for resonant excitation of standing capillary waves of relatively small wavelength. The possibility of such an effect follows from an analysis of the following system of equations

$$\begin{gathered}\rho \frac{\partial \phi}{\partial t}\Big|_{t=0} - \sigma \Delta_2 \xi = P_{el}(\mathbf{r},t) - \bar{P}_{el}(t) \\ P_{el}(\mathbf{r},t) = e\tilde{E}\, e^{i\omega t} n(\mathbf{r}), \qquad \bar{P}_{el}(t) = e\tilde{E}_\perp e^{i\omega t} n_s \\ n(\mathbf{r}) = \sum_{\mathbf{q}} (n_{\mathbf{q}} e^{i\mathbf{q}\mathbf{r}} + \text{c.c.}), \quad \overline{n(\mathbf{r})} = n_s \\ \Delta_3 \phi = 0 \qquad \frac{\partial \phi}{\partial z}\Big|_0 = -i\omega\xi\end{gathered} \tag{2}$$

$\tilde{E}_\perp$ and ω are the amplitude and frequency of the alternating electric field normal to the helium surface, $n(\mathbf{r})$ is the periodic charge density, $\mathbf{q}$ is the vector of the two-dimensional reciprocal lattice of this periodic structure, $\xi(\mathbf{r},t)$ is the amplitude of the oscillations at the free surface of helium, ρ and σ are the density and surface-tension coefficients of liquid helium, ϕ is the hydrodynamic potential, Δ_2 and Δ_3 are the two-dimensional and three-dimensional Laplace operators, and the z axis is directed into the interior of the liquid phase. The presence of $P_{el}(t) \neq 0$ leads to excitation of volume sound in the system. However, unlike the surface problem which is specified by Eqs. (2), the excitation of the volume waves contains no resonant singularities, and will not be discussed here.

The solution of (2) for $\xi(\mathbf{r},t)$ is

$$\xi(\mathbf{r},t) = e^{i\omega t} \sum_{\mathbf{q}} (\xi_{\mathbf{q}} e^{i\mathbf{q}\mathbf{r}} + \text{c.c}), \quad \xi_{\mathbf{q}} = \frac{e\tilde{E}_\perp |\mathbf{q}| n_{\mathbf{q}}}{\sigma|\mathbf{q}|^3 - \omega^2\rho}. \tag{3}$$

From (3) we see that under the conditions

$$\omega^2 = \frac{\sigma}{\rho} |\mathbf{q}|^3 \tag{3a}$$

the amplitude of the surface oscillations becomes resonantly large. For a quadratic lattice, for example when $q_x = (2\pi m/r_s)$ and $q_y = 2\pi l/r_s$, where m and l are integers, the condition (3a) takes the form

$$\omega^2 = \frac{\sigma}{\rho}\left(\frac{2\Pi}{r_s}\right)^3 (m^2 + l^2)^{3/2} \tag{3b}$$

and defines a complete set of resonant frequencies that are multiples of the numbers m and l.

The width of the resonance lines is due to the presence in the helium of interacting thermal excitations, both volume and surface, and also to the Debye-Waller factor, which is proportional to the mean-squared electron displacement $\langle u^2 \rangle$ from the equilibrium positions at the points of the Wigner lattice. It should be noted that the expression for $\langle u^2 \rangle$ in the case of a two-dimensional lattice turns out to be a quantity that diverges logarithmically when the crystal dimensions tend to infinity. However, even weak limitations on the maximum dimensions of the two-dimensional lattice can make this divergence no longer dangeous. Thus, in the case of linear dimensions $L \sim 1$ cm and $r_s \approx 10^{-4}$ cm the numerical value of $\langle u^2 \rangle$, calculated in[3], turns out to be

$$\langle u^2 \rangle \approx (2\cdot 10^{-11} + 3\cdot 10^{-10}\, T)\,\text{cm}^2. \tag{4}$$

The first term in (4) is due to the zero-point oscillations of the electrons. The second represents temperature increments. From (4) it follows that the inequality

$$\langle u^2 \rangle \ll r_s^2 \ , \tag{4a}$$

the satisfaction of which is essential for stable existence of the lattice, holds to temperatures $T \lesssim 1°$.

It is appropriate to state here that all the calculations of[3] were performed for a two-dimensional electron gas localized over an ideally flat surface of helium. Actually, when the inequality (4a) is satisfied, each electron of the lattice "presses down" on the helium surface, and by the same token increases the degree of its localization.[4] It must also be kept in mind that the amplitude of the mean-squared displacements can be greatly decreased by "constructing" the lattice of charges not out of surface electrons, but out of positive ions that are pressed against the helium surface by the liquid phase.[5] In this case $\langle u^2 \rangle$, which is inversely proportional to the mass of the charges of a given sort, is decreased by three or four orders of magnitude in comparison with its "electronic" value, in accord with the difference between the masses of the electron and the positive ion.

A concrete calculation of the shapes of the resonance lines with allowance for all the foregoing factors calls for special analysis and will be reported in a more detailed paper.

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