Polarization of short-lived states in electroexcitation reactions

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We calculated the polarization of the short-lived states in electroexcitation reactions; measurement of this polarization makes it possible to determine the relative phase of the off-diagonal matrix elements and the static moments of the short-lived states. The excitation of the $1/2^+$ level in the reaction $^7Li(e, e')^7Li^*$ is considered as an example. The main contribution is due to the Mott rescattering and to the interference of the $C2$ and $M1$ transitions, and amounts to $\sim 6\times 10^{-12}$ at the maximum. The contribution of the magnetic moments is $\sim 3\times 10^{-6}$.

When electrons are scattered by nuclei, polarization is produced in the final state. For low electron energies $\epsilon (\epsilon \ll R^{-1}$, where R is the radius of the nucleus), this polarization is determined by the charge of the nucleus and by the static moments of the transition; unlike the total cross sections, the polarization depends linearly either on the matrix elements themselves or on their ratio. Measurement of the polarization effects makes it possible to determine the relative phases of the nuclear matrix elements and appears to be the only method of measuring the static moments for short-lived states with lifetime $\lesssim 10^{-12}$ sec. Experiments of this type can be performed with existing linear electron accelerators.

The technique of calculating the polarization due to the electromagnetic interaction has been considered many times in elementary particle physics.[1] We examine the specifics of electroexcitation reactions of the type $A(e,e')A^*$. As is well known, in the case of unpolarized initial states the polarization vanishes identically if we are considering the single-photon approximation for the electroexcitation process (Fig. 1a). If, however, we admit of the possibility of two-photon or many-photon processes, then polarization of the excited nucleus I^* appears and is determined in the lowest order in α by the interference of the single-proton diagram of the Born approximation with the imaginary part of the two-photon diagram (Fig. 1b). The latter is closely related to the amplitude of the inelastic Compton effect on the nucleus. The imaginary part of this amplitude is determined by a sum over the intermediate states of the nucleus. If we use low-energy electrons, the excitation of which does not produce a continuum, then only the ground and first-excited states of the nucleus contribute to the sum over the intermediate states. The polarization is also determined (apart from the contribution of the Mott rescattering) by the static electromagnetic moments of these states. The polarization can be determined by measuring the angular correlation of the γ quanta in the $A^* \to A+\gamma$ decay, or by measuring the polarization characteristics of the ground state produced in this decay.

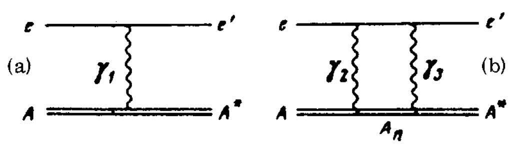

FIG. 1. Single-photon and two-photon contributions to the reaction $A(e,e')A^*$. The symbol A_n in Fig. 1b denotes an intermediate nuclear state: $A_n = A$ or $A_n = A^*$; γ_1, γ_2, and γ_3 are the photon multipolarities.

We consider the polarization of the recoil nucleus when the main electroexcitation process proceeds via an $M1$ transition, but contributions of the $C2$ and $E2$ multipoles are allowed in addition to $M1$. In this case the main contribution is made by the longitudinal $C2$ and the transverse $E2$ photons. This contribution is determined by the interference of the diagrams of Fig. 1, where γ_1, γ_2, and γ_3 denote different combinations of photons with multipolarities $M1$, $C0$, $C2$ or $M1$, $C0$, $E2$, and amounts to $\sim \alpha\epsilon(mR)^2/m \sim 10^{-3}$, where m is the nucleon mass. A contribution of the same order is due to the Mott rescattering ($\gamma_1 = M1$, γ_2, $\gamma_3 = M1$, $C0$). We note that measurements of this polarization would make it possible in principle to determine the static electromagnetic moments (magnetic, quadrupole, etc.) of a short-lived nuclear state, but the contribution of these moments is quite small. For example the contribution of the magnetic moment ($\gamma_1 = \gamma_2 = \gamma_3 = M1$) amounts in all cases to $\sim \alpha\epsilon/m \sim 10^{-5}$. The effect of the interaction of the magnetic moment of the nucleus A^* with the Coulomb field of the incident particle in the Coulomb-excitation reaction is considered in[2].

Our result for the polarization of the short-lived state takes the form

$$P = P_{MOTT} + P_C + P_E + P_M, \tag{1}$$

where P_C is the contribution of the longitudinal $C2$ photons, P_E is the contribution of the transverse $E2$ photons, and P_M is the contribution of the magnetic moments of the nuclei A and A^*. We note that by virtue of the Siegert theorem[3] we have $P_E \to 0$ as $\zeta \to 1$, where $\zeta = \epsilon_2/\epsilon_1$, and ϵ_1 and ϵ_2 are the initial and final energies of the electrons. The contribution of the Coulomb transitions is[1)]

$$P_C = \frac{Z\alpha}{5\sqrt{2}}\,\sigma(\epsilon_1 a_C)\,\frac{\sin\theta}{1+\zeta+\zeta^{-1}-z}\,[\phi_C(\zeta, z) + \zeta\phi_C(\zeta^{-1}, z)] \tag{2}$$

$$\sigma = (-)^{j_1+j_2+1}\sqrt{\frac{3(2j_2+1)}{2j_2}}\begin{Bmatrix} 1 & j_2 & j_2 \\ j_1 & j_2 & 2 \end{Bmatrix}, \tag{3}$$

where j_1 and j_2 are the spins of the ground and excited states, Z is the charge of the nucleus, $\alpha_C = \langle 1 \| C2 \| 2\rangle / \langle 1 \| M1 \| 2\rangle$, $z = \cos\theta$, and θ is the scattering angle. The contribution of the electric transitions at

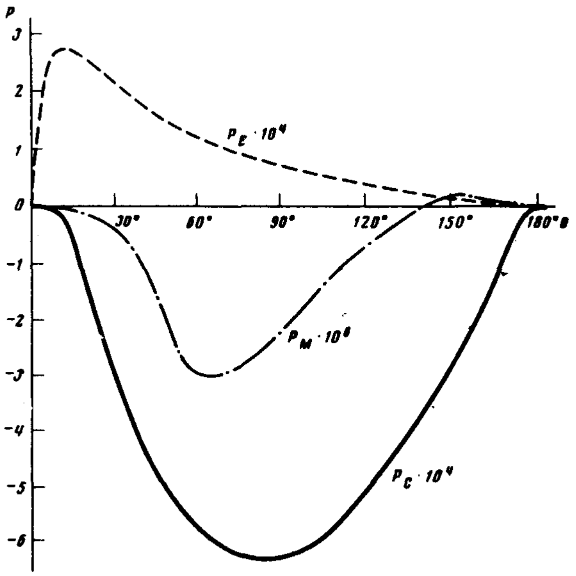

FIG. 2. Polarization of the $1/2^+$ level in the reaction $^7Li(e,e')^7Li^*$. The contributions of P_C, P_E, and P_M are shown separately.

$\epsilon \ll R^{-1}$ is again expressed in terms of α_C and is equal to

$$P_E = -\frac{3Z\alpha}{10\sqrt{2}}\sigma(\epsilon_1\alpha_C)\frac{\sin\theta}{1+\zeta+\zeta^{-1}-z}[\phi_E(\zeta z) + \zeta\phi_E(\zeta^{-1}, z)]. \quad (4)$$

We note that P_C and P_E depend linearly on the charge of the nucleus and on the electron energy.

The explicit expression for the functions $\phi_C(\zeta, z)$ and $\phi_E(\zeta, z)$ is:

$$\phi_C(\zeta, z) = \frac{1}{4} + \zeta + \frac{2+\zeta(1+z)}{1+z}\ln\frac{1-z}{2} - \frac{3}{2\zeta}\{I_1(t) + [2(1+\zeta) + \zeta(t-z)]I_2(t, z)\}, \quad (5)$$

$$\phi_E(\zeta z) = \frac{(1-\zeta)}{\zeta}\left[1 - \zeta + \frac{2+\zeta(t-z)}{1+z}\ln\frac{1-z}{2}\right] + \zeta\left\{I_1(t) - \frac{1}{2} + \frac{1}{2}(1-\zeta^{-1})^2 I_2(t, z) + [2+\xi(3-z)][I_2(t, z) - \frac{1}{1+z}\ln\frac{1-z}{2}]\right\}, \quad (6)$$

where

$$I_1(t) = \frac{1}{4}\left[2t + (t^2-1)\ln\frac{t-1}{t+1}\right],$$

$$I_2(t, z) = \frac{1}{2\sin^2\theta}\left[(1-tz)\ln\frac{t-1}{t+1} + (t-z)\ln\frac{(t-z)^2}{t^2-1}\right], \quad t = \frac{\zeta+\zeta^{-1}}{2}. \quad (7)$$

The contribution of the Mott rescattering is

$$P_{MOTT} = -\frac{Z\alpha}{2\sqrt{6}}\rho\frac{\sin\theta}{1+\zeta+\zeta^{-1}-z}(\zeta^{-1}-\zeta)\left(\frac{2}{1+z}\ln\frac{1-z}{2} + 1\right),$$

$$\rho = (-)^{j_1+j_2}\sqrt{\frac{3}{2}\frac{2j_2+1}{j_2}}\begin{Bmatrix} j_1 & 1 & j_2 \\ 1 & j_2 & 1 \end{Bmatrix}. \quad (8)$$

We note that $P_{MOTT} \to 0$ as $\zeta \to 1$.

To estimate the magnitude of the effect we consider the reaction $^7Li(e,e')\,^7Li^*$ ($3/2^+ \to 1/2^+$ transition with excitation energy $\omega = 0.478$ MeV). In this case the angular distribution of the decay γ quanta is isotropic. We choose $\epsilon_1 = 2$ MeV so as to avoid the influence of the $^4He + {}^3H$ channel, which is easily excited by $E1$ transitions.[4] In this case $\sigma = 1/\sqrt{2}$. Using the experimental values[5] of $B(C2;\uparrow)$ obtained from Coulomb excitation, and the lifetime of the $1/2^+$ level, we can estimate the parameter α_C in formula (2) and by the same token the absolute value of the polarization $P_C + P_E$. Assuming $B(C2;\uparrow) = 7e^2\ F^4$ and $\tau = 9\times10^{-4}$ sec, we obtain $|\langle 1\|C2\|2\rangle| = 5.29\ e\ F^2$ and $|\langle 1\|M1\|2\rangle| = 3.40\ (e/2m)$, whence $|\alpha_C| = 14.79$ F and $\epsilon_1\langle\alpha_C| = 0.15$. The polarization calculated with these values of the constants, under the assumption $\alpha_C > 0$, is shown in Fig. 2. The maximum value is reached at $\theta \approx 90°$ and equals -5.56×10^{-4}. The actual sign of the polarization, however, is determined by the relative phases of the reduced matrix elements. The polarization $|P_{MOTT}|$ is negative, the maximum of $|P_{MOTT}|$ is reached at $\theta \sim 20°$ and equals $\sim 6\times10^{-4}$, and $P_{MOTT}(90°) = -1.2\times10^{-4}$.

We present the result of the calculations for the contribution of the magnetic moment[2)]:

$$P_M = \frac{\alpha}{40}\left(\frac{\epsilon}{m}\right)(\lambda\mu + \lambda^*\mu^*)\frac{\sin\theta}{3-z}\left(-7 + \frac{9-5z}{1+z}\ln\frac{2}{1-z}\right), \quad (9)$$

$$\lambda = (-1)^{j_1+j_2}\sqrt{\frac{(2j_1+1)(j_1+1)}{j_1}}\sqrt{\frac{3(2j_2+1)}{2j_2}}\begin{Bmatrix} j_1 & 1 & j_2 \\ j_2 & 1 & j_1 \end{Bmatrix},$$

$$\lambda^* = (-1)^{2j_2}\frac{2j_2+1}{j_2}\sqrt{\frac{3}{2}(j_2+1)}\begin{Bmatrix} j_1 & 1 & j_2 \\ j_2 & 1 & j_2 \end{Bmatrix} + \frac{1}{j_2}\sqrt{\frac{3}{2}(j_2+1)}, \quad (10)$$

where μ is the magnetic moment of the ground state and μ^* is the magnetic moment of the excited state. The values of P_M for $\mu = \mu^* = 3.26$ are also shown in Fig. 2; the maximum of P_M is reached at $\theta \approx 60°$ and amounts to $\approx 3\times10^{-6}$.

In the derivation of (9) we used the connection between the diagonal reduced matrix element of the $M1$ transition and the magnetic moment (see, e.g.,[7]). We note that our definition of the $M1$ multipoles differs in sign from the definition used in that book.

The described mechanism for the onset of polarization leads also to the appearance of asymmetry of scattered electrons in the electroexcitation of polarized initial nuclei (there is no such asymmetry in the Born approximation). If no other polarization characteristics are measured, the cross section of the reaction is given by

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_0[1 + A(\mathbf{n}\vec{\xi}_i)],$$

where ξ_i is the polarization of the initial nucleus. The asymmetry A can again be represented in a form analogous to (1): $A = A_{MOTT} + A_C + A_E + A_M$, where A_{MOTT}, A_C, A_E, and A_M are given as before by expressions (2) and (4), (8), and (9) in which, however, it is necessary to change the values of the parameters σ, ρ, λ, and λ^*, which depend on the spins j_1 and j_2.

The details of the calculations, and also a more detailed analysis of the polarization phenomena for transitions of different multiplicities, will be considered in a separate paper.

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1) In the calculation of P_C and P_E we neglect the electron mass, but take into account the energy lost to excitation: $\zeta \neq 1$.

2) This contribution was first calculated by one of us in collaboration with L.A. Kondratyuk.[6] In formula (8) we put $\zeta = 1$.

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