Connection between the vertex constants and the electromagnetic form factors of nuclei

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It is assumed that the main contribution to the rapidly varying part of the charge form factors of nuclei at low momentum transfers q is made by amplitudes of Feynman triangle diagrams having singularities in q closest to the physical region. Under this assumption, it is possible to express the vertex constants G_{dnp} and $G_{Li^6\alpha d}$ for the decays $d \to n + p$ and $Li^6 \to \alpha + d$ in terms of the experimental charge radii of H^2 and Li^6, thus obtaining $G^2_{dnp} = 0.43$ F and $G_{Li^6\alpha d} = 0.13$ F.

The vertex constant G_{ABC} for the virtual decay of a nucleus A into fragments B and C contain important spectroscopic information. In this paper we indicate the possibility of extracting the values of G from data on the electromagnetic form factors of nuclei. We use the same definition of G_{ABC} as in[1].

We note that the charge density $\rho_A(r)$ of the nucleus A (r is the distance from the mass center of the nucleus) behaves as $r \to \infty$ like

$$\rho_A(r) \approx C_1 \exp(-2\kappa_{BC} r)/r^2, \tag{1}$$

where C_1 is a constant, $\kappa^2_{BC} = 2m_A m_C \epsilon_{ABC}/m_B$, $\epsilon_{ABC} = m_B + m_C - m_A$, m_i is the mass of the nucleus i, and the fragments B and C are chosen such that virtual decay $A \to B + C$ corresponds to the minimal value of κ_{BC}. The constant G^2_{ABC} is linearly connected with C_1. We can therefore obtain G^2_{ABC} in principle if we know the charge density $\rho_A(r)$ as $r \to \infty$. However, the values of $\rho_A(r)$ at large r are not known with acceptable accuracy.[1)] We therefore start with the nuclear charge form factor $F_A(q)$, which, unlike $\rho_A(r)$, is a quantity that can be directly measured in experiment. By definition

$$F_A(q) = \frac{1}{Z_A e}\int \rho_A(r) \exp(i\mathbf{q}\mathbf{r}) d^3r, \quad F_A(0) = 1 \tag{2}$$

($Z_A e$ – charge of nucleus A).

Using expression (1) for $\rho_A(r)$ at large r, we easily see that the singularity of $F_A(q)$ closest to the physical region ($q^2 \geqslant 0$) is the point $q^2 = q^2_\Delta = -4\kappa^2_{BC}$; at $q^2 = q^2_\Delta$, the integral (2) diverges. According to[2], q^2_Δ coincides with the position of the proper singularity of the triangle diagram of Fig. 1, where the dashed line corresponds to a virtual γ quantum with momentum $\mathbf{q}$. We can therefore assume that at small q the dependence of F_A on q is determined mainly by the amplitude of the diagram of Fig. 1; the contribution made to F_A by other more distant singularities can be approximately replaced by a constant. This assumption is more valid the closer q^2_Δ to 0 and the farther the remaining singularities.

We start with the deuteron. $F_d(q)$ is given by[3]

$$F_d(q) = f_d(q)[F_p(q) + F_n(q)], \tag{3}$$

where $F_p(q)$ and $F_n(q)$ are the charge form factors of the proton and neutron, and $f_d(q)$ is the so-called structure form factor of the deuteron. We assume that $F_d(q)$ at small q is given by

$$f_d(q) = F_{\Delta 1}(q) + C_2, \quad C_2 = 1 - F_{\Delta 1}(0), \tag{4}$$

where $F_{\Delta 1}(q)$ is the amplitude of the diagram of Fig. 1 at $A = d$, $B = n(p)$, and $C = p(n)$; in accordance with the definition (2), the vertex function $NN\gamma$ should be re-

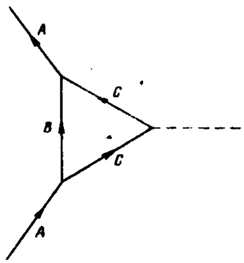

FIG. 1. Feynman triangle diagram for the charge form factor.

placed by unity. According to[2] we have[2)]

$$F_{\Delta 1}(q) = (m_N^2 G_{dnp}^2 / 2\pi q) \operatorname{arctg}(q/\sqrt{16\epsilon_{dnp} m_N}). \quad (5)$$

Expanding expression (4) in powers of q^2 and comparing the obtained expansion with the known formula $f_d(q) = 1 - (1/6)\langle \tilde{r}_d^2 \rangle q^2 + O(q^4)$, we obtain

$$G_{dnp}^2 = a < \tilde{r}_d^2 >, \quad a = 64\pi m_N (\epsilon_{dnp}/m_N)^{3/2}. \quad (6)$$

Here $\langle \tilde{r}_d^2 \rangle$ is connected with the charge radius of the deuteron $\langle r_d^2 \rangle^{1/2}$ by the relation $\langle r_d^2 \rangle = \langle \tilde{r}_d^2 \rangle + \langle r_p^2 \rangle - \langle r_n^2 \rangle$, where $\langle r_N^2 \rangle^{1/2}$ $(N = p, n)$ is the rms radius of the nucleus. Using the experimental value $\langle \tilde{r}_d^2 \rangle^{1/2} = 1.9635 \pm 0.0045$ F, [4] We obtain from (6) the value $G_{dnp}^2 = 0.426 \pm 0.002$ F. This result is in splendid agreement with the value $G_{dnp}^2 = 0.43 \pm 0.01$ F obtained from calculations with realistic NN potentials.

A more interesting example is the nucleus Li^6. In this case the closest singularity $q_\Delta^2 = -0.85$ F^{-2} corresponds to the diagram of Fig. 1 with $B = He^4$ and $C = d$. Such a diagram arises naturally in the cluster model of the Li^6 nucleus, in which its wave function is given by $\psi = \phi_\alpha \phi_d \chi(r_{\alpha d})$, where ϕ_α and ϕ_d are the internal wave functions of the α and d clusters, and χ is the wave function of the relative function of the clusters. Using this wave function, we easily obtain from (2)

$$F_{Li^6}(q) = \frac{1}{3} f_{Li^6}(q) F_d(q) + \frac{2}{3} f_{Li^6}(q/2) F_\alpha(q), \quad (7)$$

where $F_d(q)$ and $F_\alpha(q)$ are the charge form factors of the d and α clusters, and $F_{Li^6}(q) = \int |\chi(r_{\alpha d})|^2 \exp(i2\mathbf{q}\cdot\mathbf{r}_{\alpha d}/3)\, d^3 r_{\alpha d}$ is the structure form factor of Li^6. Just as in the case of the deuteron we put at small q

$$f_{Li^6}(q) = F_{\Delta 2}(q) + C_3, \quad C_3 = 1 - F_{\Delta 2}(0),$$
$$F_{\Delta 2}(q) = \frac{8m_N^2}{3\pi} \frac{G^2}{q} \operatorname{arctg} \frac{q}{\sqrt{24 m_N \epsilon_{Li^6 \alpha d}}}. \quad (8)$$

Here $F_{\Delta 2}(q)$ is the amplitude of the diagram of Fig. 1 with $A = Li^6$, $B = \alpha$ and $C = d$; $G^2 \equiv G_{Li^6/\alpha d}^2$. Substituting (8) in (7), expanding $f_{Li^6}(q)$, $F_d(q)$, and $F_\alpha(q)$ in powers of q^2, and equating the coefficients of q^2, we obtain

$$G^2 = b\left(< r_{Li^6}^2 > - \frac{1}{3} < r_d^2 > - \frac{2}{3} < r_\alpha^2 >\right),$$
$$b = (3\pi m_N/8)(24\epsilon_{Li^6 \alpha d}/m_N)^{3/2}. \quad (9)$$

Using the experimental values $\langle r_{Li^6}^2 \rangle^{1/2} = 2.54 \pm 0.06$ F (the mean value of the results of[5-7]), $\langle r_d^2 \rangle^{1/2} = 2.095 \pm 0.006$ F[4], and $\langle r_\alpha^2 \rangle^{1/2} = 1.69 \pm 0.02$ F,[8] we obtain $G^2 = 0.13 \pm 0.02$ F. The form factor $F_{Li^6}(q)$ calculated from formulas (7) and (8) at $G^2 = 0.13$ F and using the experimental values of $F_d(q)$ and $F_\alpha(q)$ from[4] and[8] is shown in Fig. 2. The experimental points were taken from[7].

We note that only the order of magnitude of the constant G^2 for Li^6 is known. An analysis of different reactions within the framework of the peripheral model[9] leads to values $G^2 = 0.064$ F (the reaction $Li^6(p, He^3)He^4$), $G^2 = 0.054$ F (the reaction $Li^6(\alpha, Li^6)He^4$)

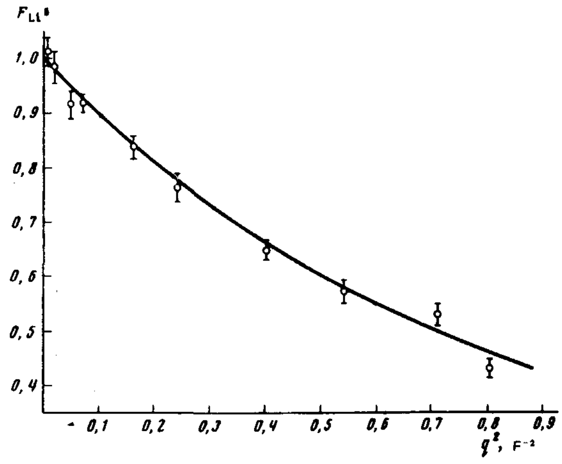

FIG. 2. Comparison of theory with experiment for the charge form factor of the nucleus Li^6. The theoretical curve was constructed at $G_{Li^6 \alpha d}^2 = 0.13$ F. The experimental points were taken from[7].

and $G^2 = 0.22$ F (the reaction $Li^6(d, Li^6)H^2$). The dispersion K-matrix approach to the reaction $Li^6(\alpha, Li^6)He^4$ yields $G^2 = 0.050$ F.[10] The value $G^2 = 0.13$ F obtained by us agrees with the value[3)] 0.126 F that follows from the result of an analysis of $d\alpha$ scattering within the framework of the N/D method.[11]

The proposed method can be used also for heavier nuclei. Using the data on inelastic electromagnetic form factors, we can obtain information on the vertex constant for excited states of nuclei.

1)Conversely, if G_{ABC} is known from other sources, then formula (1) can be used to obtain information on $\rho_A(r)$ at large r.

2)We neglect the small admixture of D-states in the deuteron.

3)This figure was obtained by A. G. Baryshnikov.

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