Self-oscillating instability of fast-flow lasers using unstable resonators

Yu. A. Dreĭzin and A. M. Dykhne

(Submitted April 26, 1974)
ZhETF Pis. Red. 19, 718–722 (June 20, 1974)

We investigate the stability of the stationary lasing regime in a fast-flow gas laser with an unstable resonator. The radiation in the resonator is described in the geometrical-optics approximation, and the evolution of the gain K of the active medium is described by the model-dependent equation (2). It is shown theoretically that the stationary lasing regime is unstable. As a result of the disruption of the stationary lasing, a pulsating-lasing regime sets in (with a pulsation period of the order of the time of flow of the gas through the resonator).

1. The so-called unstable resonators[1,2] have recently gained wide use in laser technology. The opinion has been advanced in a number of papers, that these resonators are universal and can be used not only in solid-state but also in fast-flow gas lasers (electric-discharge, gas dynamic, etc.). We wish to show that a combination of an unstable resonator with a rapidly flowing active medium can lead to unique self-oscillation instability of laser operation.

2. The laser diagram is shown in Fig. 1. The field in an unstable resonator, as is well known, [2] can be represented in the form of two spherical (cylindrical) waves, the amplitudes of which vary slowly in space. If the amplitudes vary little over the diffraction dimension ($\sqrt{L\lambda}$), then each of the waves can be described in terms of geometrical optics:

$$\frac{1}{v_{1,2}^d}\frac{\partial}{\partial v_{1,2}}(v_{1,2}^d I_{1,2}) = K I_{1,2}. \qquad (1)$$

Here I is the intensity of the light, v is the distance to the center of the wave, $d=2$ for spherical mirrors and $d=1$ for cylindrical mirrors, and K is the local gain. In addition to Eqs. (1), we should write down also boundary conditions on the mirrors, corresponding to the transformation on the waves into one another upon reflection. Equations (1) are valid if K depends on the time but varies little during the resonator time L/c. Equations (1) should be supplemented by equations for the active medium, which makes it possible to determine K and which describe the processes of transfer and relaxation (including radiative relaxation) of the excitations in the particular medium. We shall use the simplest equations for K, which take these processes into account qualitatively:

$$\frac{\partial K}{\partial t} + v\frac{\partial K}{\partial x} = -\delta I K - \gamma K. \qquad (2)$$

Here v is the stream velocity, $I = I_1 + I_2$, and the constants δ and γ describe the radiative and collision relaxation in the resonator zone. (It is assumed that there is no pumping in the resonator zone.) At a given K, Eq. (1) has a nonzero solution that is finite on the optical axis only if the generation condition

$$\exp\{2\int_a^b K dl\} = M \qquad (3)$$

is satisfied, where M is the resonator gain, equal to the product of the gains of the mirrors, $M = (v_{1a}v_{2b}/v_{1b}v_{2a})^d$, and $\int_a^b K dl$ is taken along the optical axis. When the generation conditions (3) are satisfied, Eqs. (1) for I have a solution that is finite on the optical axis and is determined apart from a factor. This factor is "chosen" in such a way that the condition (3) is satisfied at all instants of time on the resonator axis as a result of the burnup of the medium. If this condition ceases to be satisfied, and the gain on the axis becomes smaller than the amplification coefficient, the light in the resonator is extinguished.

3. Let us describe qualitatively how the stationary regime of the considered laser can become unstable. Let, e.g., a positive fluctuation of the gain take place at the entry into the resonator. During the time that this fluctuation moves towards the axis, an increased radiation (in comparison with a stationary one) will be observed at the entrance. This increased-power radiation, causes a larger burnup of the active medium in the upstream part of the resonator. Thus, during the time of travel of the gas through the resonator, a negative fluctuation of K will be observed at the entrance. Such a self-oscillating process will grow if the secondary fluctuation is larger in magnitude than the initial fluctuation. We shall show in Sec. 4 that this is indeed the case if K varies in accordance with (2).

4. The system (1)—(3) is still quite complicated.

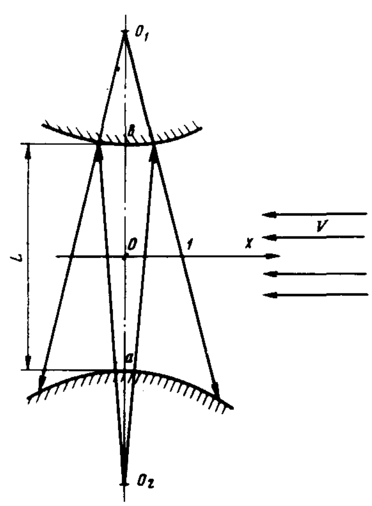

FIG. 1. Diagram of generator with gas flow.

Considerable simplification can be obtained in a case of a weakly amplifying medium, for which $\int K dl \ll 1$. To obtain lasing in this case, it is necessary to use a resonator with $M - 1 \ll 1$. These inequalities enable us to neglect the change of the gain in the direction of the optical axis, and to obtain for I an equation that is averaged over this direction:

$$\frac{1}{\rho^{d-1}} \frac{\partial}{\partial \rho} (\rho^d I) = \frac{2KI}{M-1}, \tag{4}$$

where ρ is the distance from the optical axis. Let us consider first a resonator with cylindrical mirrors ($d = 1$, $\rho \equiv x$). Making a scale transformation of the quantities x, t, I, and K in Eqs. (2) and (4), and using the same notation for the dimensionless variables, we obtain

$$x \frac{\partial I}{\partial x} = (K - 1) I, \tag{5}$$

$$\frac{\partial K}{\partial t} - \frac{\partial K}{\partial x} = -KT - \gamma K. \tag{6}$$

These equations must be solved under the conditions $K = K_{in}$ at the entrance (at $x = 1$) and $K = 1$ at the center (at $x = 0$). The last equation expresses the condition (3) in dimensionless variables. Equations (5) and (6) admit of the solutions $K_0(x)$ and $I_0(x)$ corresponding to the stationary regime of the laser. Let us investigate the stability of this regime. Representing K and I in the form $K = K_0(x)[1 + \beta(x, t)]$ and $I = I_0(x)[1 + \alpha(x, t)]$ and eliminating α from the linearized equations for α and β, we obtain

$$\frac{\partial}{\partial x}\left[I_0^{-1}\left(\frac{\partial \beta}{\partial x} - \frac{\partial \beta}{\partial t}\right)\right] = \frac{K_0 \beta}{x}. \tag{7}$$

Multiplying (7) by $\partial\beta/\partial t$ and integrating with respect to x from zero to unity, we obtain, with allowance for the zero boundary conditions for β,

$$\frac{\partial}{\partial t} \int_0^1 \left[\frac{K_0}{x} \beta^2 + I_0^{-1}\left(\frac{\partial \beta}{dx}\right)^2\right] dx = \int_0^1 I_0^{-2} \frac{dI_0}{dx}\left(\frac{\partial \beta}{\partial t}\right)^2 dx. \tag{8}$$

Recognizing that $K_0(x) \geq 1$ in stationary lasing, we find from (6) that $dI_0/dx > 0$. The integral in the left-hand side of (8), and with it also the perturbation β, increases with time, and the stationary regime turns out to be unstable.

5. In the case of spherical mirrors, even the stationary problem becomes two-dimensional. Nonetheless, an analysis of the stability can be carried out also in this case. The point is that on the x axis Eqs. (2) and (4) can be solved autonomously, without constructing the solution in the entire resonator. An investigation based on this remark is carried out in the same manner as in Sec. 4, and leads to the same result.

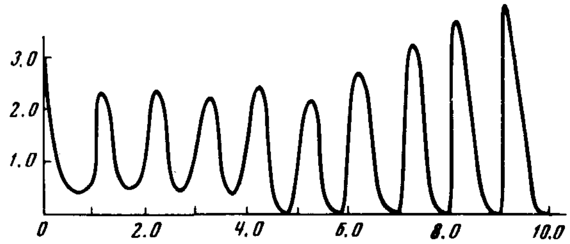

FIG. 2. Computer-calculated diagram of radiation at the exit from the resonator.

6. As the result of the collapse of the stationary generation, a "pulsating" regime is apparently established. Its establishment can be traced quite clearly if K at the entrance to the resonator is changed jumpwise, when the laser is started, from zero to K_{in}. In this case the radiation first appears when the front of the active gas arrives at the resonator axis. Since K should decrease from K_{in} to unity (we neglect for simplicity the relaxation) within a short time on the order of the time of travel through the width of the front, powerful radiation will flare up in the resonator. So long as $K \geq 1$ in the resonator, the radiation is amplified away from the axis [see (5)]. This causes the active gas to burn out everywhere more strongly than on the axis, i. e., K becomes smaller than unity, the radiation is extinguished and stays extinguished so long as the burned-up gas remains on the resonator axis. A new flash occurs after a time equal to the time of travel of the gas through the resonator, when "fresh" gas arrives at the axis.

7. Let us discuss qualitatively an effect not accounted for in our model equation (2), namely the saturation of the gain at high radiation intensities, $I \gtrsim I_0$, where I_0 is the saturation power. To take this affect approximately into account, it is necessary to replace KI in the right-hand sides of (1) and (2) by $KI/(1 + I/I_0)$. [3] It is easy to show that if $I \gg I_0$ in the stationary regime, that this regime is stable. However, even if $I \ll I_0$ in the stationary regime, allowance for I_0 is important for the analysis of the flashes described in Sec. 6. In particular, I_0 can determine the duration τ of the flashes (as $I_0 \to \infty$ and neglecting the spreading of the front of the "fresh" gas as $\tau \to 0$). A computer solution of Eqs. (5) and (6) confirm this conclusion. Figure 2 shows a plot of the radiation at the exit from the resonator (at $K_{in} = 2$ and $I_0 = 10$), illustrating the collapse of the stationary regime and the onset of the pulsating regime.

We are grateful to L. A. Bol'shov, A. A. Vedenov, E. P. Velikhov, and A. P. Napartovich for useful discussions, and to Yu. M. Panchenko for numerical calculations.

[1]A. E. Siegman, Proc. IEEE 53, 277 (1965).
[2]Yu. Anan'ev, Usp. Fiz. Nauk 103, 705 (1965) [Sov. Phys.-Usp. 14, 197 (1965)].
[3]A. V. Eletskiĭ and B. M. Smirnov, Gazovye lazery (Gas Lasers), Moscow, Atomizadat, 1971.