Strong Langmuir turbulence, turbulent plasma heating by electron beams, and laser compression of matter

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(Submitted April 26, 1974)
ZhETF Pis. Red. **19**, 729–733 (June 20, 1974)

The laws of turbulent heating of a plasma by electron beams and by light are predicted on the basis of the soliton model of Langmuir turbulence, and conditions are established for the realizability of ultrastrong compression of matter.

1. The most powerful of the probable channels through which energy is transferred from intense electron beams or from electromagnetic waves to thermal plasma is Langmuir turbulence. This is confirmed, in particular, by one-dimensional numerical experiments, in which the Langmuir turbulence is produced in the plasma as the result of parametric instabilities excited by the electromagnetic wave[1] or by an electron beam.[2] As a result of the great decrease in scale, a quasi-stationary state is established with $|E_k|^2 \sim k^{-2}$ and with a turbulent-energy flux w from the generation region $k_0 = \omega_p/c$ to $k_{\max} \approx \omega_p/v_{Te} \equiv r_D^{-1}$, where the plasma oscillations are absorbed through linear Landau damping, and a high-energy electronic component is produced.

These results can be explained by the soliton model of turbulence.[1-5] Solitons are produced as a result of modulation instability of homogeneous Langmuir turbulence at $\overline{w}/nT > (k_0 r_D)^2$, and constitute bunches of plasma oscillations trapped in plasma-rarefaction regions produced by the high-frequency field pressure. The formation of solitons is energywise favored, since the frequency of the Langmuir "quanta" decreases in this case and the released energy is more than sufficient to push the plasma asunder. The energy of the oscillations contained in the soliton is determined by the soliton volume, and the minimum dimension of the soliton depends on the field amplitude

$$\int \frac{E^2}{4\pi} d^3r \approx \begin{cases} 3nTr_D^2 l^{-1} \\ 3nTr_D^2 l \end{cases}, \qquad l \approx r_D \left(\frac{12\pi nT}{E^2} \right)^{1/2}, \tag{1}$$

in the one-dimensional and three-dimensional cases, respectively. As the result of collisions, the solitons can either merge or split up. In the one-dimensional case, the merging of two solitons produces a soliton of smaller width, and in the three-dimensional case solitons with small scales appear when larger ones are subdivided. Since the average distance between solitons with $l/r_D \ll (w/nT)^{1/2}$ is much larger than their dimension, pair collisions will predominate. Theoretical considerations and a numerical experiment for the one-dimensional case show that solitons of comparable amplitudes interact, and the characteristic soliton velocity is $v_{T_i} < v < C_s$.[5] In the three-dimensional case, therefore, owing to the splittings that occur with frequency $\sim \pi C_s l^2 N_l$, the energy flowing into the absorption region is

$$q \approx \pi n T r_D^2 C_s l^3 N_l^2 = -\frac{d}{dt} n'T'. \tag{2}$$

Here n' and T' are the density and temperature of the hot component of the electrons, and N_l is the number of solitons with scale l per unit volume.

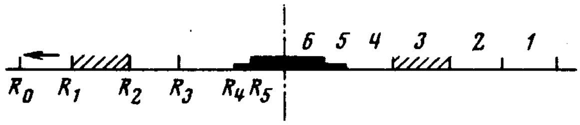
An electron traveling through a soliton within a time shorter than the half-period π/ω_p of field variation, changes its velocity on the average by an amount Δv, where $mv\Delta v = eEl \approx T$. As the result of multiple collisions with frequency $\pi l^2 vN$ $(l < v/\omega_p)$, the electrons acquire energy in accordance with the equation

$$\frac{df'}{dt} = \pi \frac{\partial}{\partial v} \frac{v_{T_e}^4}{v^2} l^2 vN\left(l < \frac{v}{\omega_p}\right) \frac{\partial f'}{\partial v} \approx \pi \frac{\partial}{\partial v} \frac{v_{T_e}^4}{v} \left(\frac{\dot{q} v}{c_s n T r_D v_{T_e}} \right)^{1/2} \frac{\partial f'}{\partial v}. \tag{3}$$

2. Turbulent heating of plasma by electron beams (n_b and v_b are the density and velocity of the beam particles). For this problem, under steady-state conditions, process, q should be taken to mean the power of generation of Langmuir energy by the beam, $q = 2\gamma_b w_b$, where γ_b and w_b are the growth rate and density of the oscillation energy in the region of unstable wave numbers. Nonlinear scattering of oscillations by ions limit w_b to the level (see[6])

$$\frac{w_b}{nT} < \frac{\gamma_b}{\omega_p} \frac{T_e}{T_i}\left(1 + \frac{T_e}{T_i} \frac{M}{m} \frac{v_{T_e}^2}{v_b^2}\right), \qquad \frac{\gamma_b}{\omega_p} \geqslant \frac{n_b}{n} \frac{m}{m_b}. \tag{4}$$

Although there exists a powerful mechanism of turbulent heating, it does not transform the mean energy into the



energy of the small number of high-energy electrons. After passing through the region of beam deceleration, $\lambda = m_b n_b v_b^2/2\gamma_b w_b$, the plasma electrons can acquire an energy given, in accordance with Eq. (3) and the estimates (4), by

$$T' = m_b v_b^2 \left[\left(\frac{M}{m}\right)^{1/2} \frac{v'_T v_{T_e}}{v_b^2} \frac{\omega_p nT}{\gamma_b w_b} \frac{n_b^2 m^2}{n m_b^2} \right]^{1/4}$$
$$\approx m_b v_b^2 \left[\frac{\left(\frac{M}{m}\right)^{1/2} \frac{v'_T v_{T_e}}{v_b^2} \frac{T_i}{T_e}}{1 + \frac{T_e}{T_i} \frac{M}{m} \frac{v_{T_e}^2}{v_b^2}} \right]^{1/4}, \tag{5}$$

i.e., in practice the temperature of the energetic component of the electrons is close to or higher than the energy of the beam particles, if the electrons pass through the heating region several times.

3. Let us examine qualitatively the process of spherical compression of matter by laser energy, under the assumption that the plasma heating process is described by Eqs. (2) and (3). The figure shows the arrangement of the characteristic regions in this process. The light-beam energy is absorbed as the result of parametric instabilities in region 3, where the frequency of the light is $f \leqslant \omega_p/2\pi$. We do not consider here the ratio of the absorbed and reflected light, and are concerned only with the quantity $Q = qV_3$, which is the power pumped into the plasma oscillations. The particles from the "tail" of the Maxwellian distribution, which become turbulently heated, acquire energy by multiple passage through region 3 and lose energy as the result of Coulomb losses, principally when they penetrate into the dense layer of matter 5. Collision with a dense nucleus is probable after $(R_0/R_4)^2$ passages of the particle through the heating region 3, so that the energy acquired by the particle during its lifetime is given, in accordance with Eq. (3), by

$$T' \approx \frac{R_0}{R_4} T_e \left[\frac{Q R_2^2}{V_3 C_s n_e T_e r_D} \left(\frac{T'}{T_e}\right)^{1/2} \right]^{1/2}. \tag{6}$$

The hot electrons escaping beyond the plasma radius R_0 produce an electric field that drags the ions with them. Region 1 is a hot plasma expanding at an average rate $(T'/M)^{1/2}$ with density n'. As the results of many reflections from the surface R_0, the hot component of the electrons becomes isotropic and its temperature and density become approximately constant in the volume from R_0 to R_5. Therefore the change of the plasma electron temperature in this volume will be described by the usual equation for T_e with a homogeneous volume source Q/n, where N is the total number of electrons in the region from R_0 to R_5. This equation together with the gas-dynamic equations for the plasma and with expression

(7) for the penetration depth determines the dynamics of the compression of the substance.

Fast electrons penetrating into region 5 heat this region and produce an excess pressure, which compresses the matter in region 6. In the steady state, the depth of penetration into a substance with atomic number Z should be comparable with R_5

$$\int_{R_5}^{2R_5} (Z+1) n_e \, dr \approx 10^{13} T'^2 \text{ eV} \approx 10^{13} T_e^2(\text{eV}) \left(\frac{R_0}{2R_5}\right)^2 \left(\frac{QR_2^2}{N_3 C_s T_e r_D}\right)^{1/4}. \quad (7)$$

The considered compression region should differ from the usual thermal regime, if one can neglect the heat transfer from region 5 and region 6 by electronic thermal conductivity. Then $T_e = \int (Q/N)\, dt$.

As is well known, the largest compression of matter in the region 6 can be obtained under conditions when no shock waves are produced in region 6. In this regime we have $dR_5/dt = (\gamma p_6/\rho_6)^{1/2} \sim 1/R_5$, where $R_5 \sim \sqrt{t}$ (the time is reckoned backward from the instant of maximum compression). To satisfy this condition, relation (7), and the approximate equality of the pressures in regions 5 and 6, it is necessary that the heating power Q vary in accordance with a definite law. During the latest stages of the compression was can neglect the changes of the dimensions R_0 and R_2 and of the particle numbers N_3 and N. We therefore should have

$$Q \sim t^{-6/5}. \quad (8)$$

This law differs strongly from the law governing the power supply in the case of pure thermal compression, $Q \sim t^{-2}$. To have equal values of Q at the end of the process, at $t = t_0$, it is necessary to expand five times more energy on the compression in the regime under consideration. This is the consequence of the fact that the temperature of the hot zone $R > R_5$ varies weakly, $T \sim t^{-1/5}$, and the plasma ejected from region 5 during the initial stage of the process is too hot in comparison with the central region. This lowers the efficiency of energy utilization.

Obviously, in order for the compression to be possible, it is necessary to satisfy the condition (7) during the last stage, when $R_5 \to R_{\min}$ and $Z_5 T_5 \to Z_6 T_6$. This is possible if

$$N_6 > 10^{13} \frac{Z_6^3}{Z_5^4} R_0^2 \left(\frac{Q_{max} R_2^2}{C_s N_3 T_5 r_D}\right)^{1/4} T_6^2(\text{eV}), \quad (9)$$

or in other words

$$N_{\text{atom}} > 10^{13} \frac{Z_6^2}{Z_5^4} R_0^2 \left(\frac{NR_2^2}{N_3 R_{min} r_D}\right)^{1/4} T_6^2(\text{eV}),$$

The numerical factor in the parentheses is $\sim 10^6$. At $R_0 = 2R_2 = 10^{-1}$ cm, $T_6 = 3 \times 10^3$ eV, $R_2/R_{\min} = 30$ we find that it is necessary to compress $10^{21} Z_6^2/Z_5^4$ atoms. To this end, at an efficiency of 10%, the minimum energy required (in Joules) is $10^7 Z_6^2/Z_5^4$.

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