Description of low-energy πN scattering in a nonlinear chiral Lagrangian

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(Submitted May 5, 1974)
ZhETF Pis. Red. **19**, 772–775 (June 20, 1974)

A satisfactory description of the energy variation of the πN scattering s and p waves is obtained in the region up to 400 MeV of kinetic laboratory energy in the single-loop approximation of the nonlinear chiral Lagrangian within the framework of Weinberg's model.

Recently Lehmann[1] obtained very interesting results on the description of the $\pi\pi$ scattering phase shifts by taking into account single-loop diagrams within the framework of the nonlinear chiral-invariant $SU(2)\times SU(2)$ Lagrangian. It is even of greater interest to perform a similar analysis of the πN scattering phase shift.

In addition to the tree-approximation diagrams, which were calculated in particular by Serebryakov and Shirkov,[2] we take into account also single-loop diagrams of the type shown in the figure, with the framework of Weinberg's chiral-invariant $SU(2)\times SU(2)$ Lagrangian.[2] These diagrams, as shown by our estimates, make the main contribution to the fourth order in the coupling constant. From the diagrams of the type shown in the figure we calculate the imaginary part of the amplitude, while the real part is reconstructed with the aid of one-dimensional dispersion relations for the invariant amplitudes $A^{\pm}$ and $B^{\pm}$. It becomes necessary here to perform two subtractions. Recognizing that in the first approximation the coupling constants are renormalized qualitites, we obtain a representation that depends on four subtraction parameters a, b, c, and d.

To separate the partial amplitudes, we use the method of combining the invariant amplitudes for the forward and backward scattering angles. The resultant expressions turned out to be crossing-symmetrical in the energy in the c. m. s.

These relations serve as a basis for obtaining the partial amplitudes. The parameters a, b, c, and d are determined completely from the s waves. Thus, it becomes possible to obtain a description of the resonant p waves without introducing arbitrary parameters.

For the s^- wave in fourth order we obtain the following expression

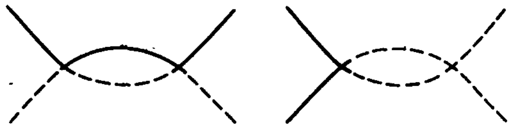
$$\mathrm{Re}S^- = \frac{f^2}{2\pi}\left[-M^2\omega b - q^2\omega d + \frac{2}{3}q^2\omega\ln\omega\right]. \tag{1}$$

Comparing (1) with the expression for $\mathrm{Re}S^-$ calculated in second order (see, e. g., [2])

$$\mathrm{Re}S^- = f^2\omega, \tag{2}$$

we see that these expressions can be regarded as the first two terms of the expansion of the s-wave amplitude

$$\mathrm{Re}S^- = \frac{f^2\omega}{1+\frac{M^2 b}{2\pi}+\frac{f^2q^2}{2\pi}\left[d-\frac{2}{3}\ln\omega\right]}. \tag{3}$$



The espression (3) coincides exactly, if $b=0$, with the expression for the s^- wave obtained in the dispersion approach by starting with allowance for the ρ-meson exchange (see, e. g., [2,3]). The constant d is then expressed in terms of the ρ-meson mass.

A similar analysis of the second and fourth orders for the s^+ waves leads to the expression

$$\mathrm{Re}S^+ = -\frac{2f^2q^2}{M\left\{1+\frac{f^2}{2\pi}[(c+4)\omega^2-1]\right\}}, \tag{4}$$

which also coincides with the expression obtained within the framework of the dispersion approach by taking into account σ-meson exchange.[2,4] The absence from (4) of terms of order M calls for satisfaction of the condition $2c+a+4=0$. Obviously, c should be expressed in terms of the σ-meson mass.

Thus, two out of the four arbitrary constants a, b, c, and d are determined from considerations of compatibility with the dispersion approach ($b=0$, $a=-20$ to -10), whereas the remaining two, c and d, are connected with the physical masses of the resonances and determine completely the behavior of the s waves. Assuming the coupling constant f^2 as firmly established by now ($f^2 = 0.08$), we see that the s waves are determined by half as many parameters as, say, in the case of a linear Lagrangian. The important result is that by defining these constants in this manner we can describe the entire aggregate of the p waves without introducing any new parameters at all.

We consider, as before, the first two terms of the expansion for the p_{33} wave. Reconstricting the amplitude from these two terms, we obtain

$$\frac{4}{3}f^2\omega^{-1}q^3\,\mathrm{ctg}\,\delta_{33} = 1 - \frac{f^2\omega}{8\pi}\left[-aM + \omega\left(2+c-d+\frac{2}{3}\ln\omega\right)\right]. \tag{5}$$

In similar fashion we obtain an expression for the remaining p waves.

The expressions obtained in this manner for the s waves describe well the experimental data at $d=10$ and $c=3$–8, corresponding in the linear model to $t_\rho \sim 30\mu^2$ and $t_\sigma \sim 44-26\mu^2$.

The "small" waves δ_{13} and δ_{31} agree well with the experimental data up to 400 MeV. The δ_{33} phase also describes well the experimental data up to ~400 MeV and contains a resonance ($\omega \approx 2$). We note that unlike the effective-radius theory, we obtain here a certain downward "bending" of the curve ($\omega^{-1}q^3 \cot\delta_{33}$), in agreement with the experimental data. Finally, the δ_{11} phase agrees only qualitatively with experiment.

Thus, our results show that even allowance for the simplest loop diagrams yields a good description of the low-energy πN scattering with the aid of a minimum number of parameters, namely the coupling constant f^2 and two constants connected with the ρ- and σ-meson masses. Starting from the minimum number of fields—pion field and nucleon field—it becomes possible to reconstruct the amplitude of the πN scattering, which contains low-energy resonances in the corresponding p waves as well as closest singularities in the annihilation channel of the s waves. One can hope that the use of the superpropagator method of regularization for nonpolynomial Lagrangians of the chiral type will make it possible to reduce the number of these parameters, as it was possible in $\pi\pi$ scattering.[5,6]

The authors are deeply grateful to D. V. Shirkov, V. A. Meshcheryakov, A. V. Efremov, P. S. Isaev, and also V. I. Zhuravlev and V. N. Pervushin for valuable discussions.

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