Conformal group and chemical affinity

A. I. Fet

Institute of Inorganic Chemistry, Siberian Division, USSR Academy of Sciences
(Submitted May 16, 1974)
ZhETF Pis. Red. **20**, 24–26 (July 5, 1974)

1. A group description of the system of chemical elements was proposed in[5], where the symmetry group was chosen to be the universal covering group $SO(4)$. In the same reference, the representation space was chosen to be the space $\vec{\mathcal{H}}$ of the two-component Fock functions $\psi_i(\xi^1, \xi^2, \xi^3, \xi^4)$, $i=1$ or 2, $\Sigma(\xi^\alpha)^2=1$. The conformal group was used for this purpose in[2].[1)]

We choose the symmetry group $G=SU(2)\times\tilde{SO}(4,2)$, where $SO(4,2)$ is the universal covering group of the conformal group. The Lie algebra AG is generated by 18 Yao generators ([6], I, pp. 1933–1934) J_α, K_α, $\mathcal{P}_\alpha$, Q_α, S_α, and T_α ($\alpha=1,2,3$), which are connected by three relations, and also by the generators s_α of the $SU(2)$ spin group. Representing s_α by Pauli matrices, and the Yao generators by operators of the representation $\mathcal{S}^+$ ([6], II, p. 1625), we obtain the irreducible unitary representation η of group G in space $\vec{\mathcal{H}}$. The representation η specifies the *Coulomb system* (cf. [5]).

2. We consider in G the subgroups $SU(2)_C$ and $SU(2)_M$, which cover the groups of the proper rotations of the subspaces (ξ^1, ξ^3, ξ^4), (ξ^1, ξ^2, ξ^3). In $\vec{\mathcal{H}}$ we choose the basis C of the eigenvectors $|n, \lambda, \mu, s_3\rangle$ of the operators $R_0=\mathcal{P}_0+Q_0=S_0+T_0$, C_C, $N_1=J_1+K_1$, and s_3, where C_C is the Casimir operator of the subgroup $SU(2)_C$, and the corresponding eigenvalues are n, $\lambda(\lambda+1)$, μ, and s_3. A similar basis $|n, l, m, s_3\rangle$ is constructed for $SU(2)_M$.

The "chemical" symmetry breaking operator A (the atomic-number operator) has eigenvalues

$$A=\frac{1}{6}d(d^2-1)+\frac{(d+1)^2}{2}-\kappa(d)\frac{1}{2}(d+1)$$
$$-2(\lambda^2+1)+2\mu+s_3+\frac{3}{2}, \qquad (1)$$

where

$$d=n+\lambda, \qquad \kappa(d)=\begin{cases}0 & \text{for odd } d,\\ 1 & \text{for even } d.\end{cases}$$

The eigenvalues of A (all simple) are $1, 2, \ldots$. The vector $|n, \lambda, \mu, s_3\rangle$ is regarded as the Coulomb-system state corresponding to the atom with atomic number A.[1]

The basis M for the subgroup $SU(2)_M$ and the "mechanical" symmetry breaking A' operator are constructed analogously. It is assumed that this breaking is small in comparison with the chemical one; cf. the hypercharge and charge mass splitting in $SU(3)$ theory.

3. With this arrangement of the elements it can be noted that the chemical analogs are arranged horizontally, i.e., they differ only in the value of n. This fact can be explained by using the analogy with $SU(6)$ theory. In the meson 35-plet and the baryon 56-plet (see, e.g., [4], Secs. 15.1A and 15.2A), the closest analogs are "particles" that differ only in the spin projection J_3 [in $SU(6)$-multiplets they are regarded as different states of the 35- and 56-plet]. The operators $J_\pm=J_1\pm iJ_2$ belonging to the complex envelope of the Lie algebra of the $SU(6)$ group, transform these "particles" into one another. These operators commute with the operators of the Lie algebra of the subgroup $SU(3)$. In exactly the same manner, to find the chemical analogs it is necessary to use operators from the complex envelope of the Lie algebra AG, which commute with the operators of the subalgebra $ASU(2)_C$. These operators are

$$C_\pm=\mathcal{P}_\pm+Q_\pm(\mathcal{P}_\pm=\mathcal{P}_1\pm i\mathcal{P}_2,\ Q_\pm=Q_1\pm iQ_2). \qquad (2)$$

Since $[R_0, C_\pm]=\pm C_\pm$, the operators (2) change the quantum number n by ± 1, without changing λ, μ, or s_3; we call them the chemical affinity operators.

4. The operators A and A' do not permute. The bases C and M do not contain common vectors. If we "remove" formally the chemical symmetry breaking, leaving only the "mechanical" one, then the Coulomb system should be identified with the hydrogen atom, which is capable of being in the energy states $|n, l, m, s_3\rangle$ (cf. [1,3]).

[1)]When this paper was being readied for press, we learned that the conformal group was applied to this problem, simultaneously with[2], by Barut (Structure of Matter, Rutherford Centennial Symposium, New Zealand, 1972).

[1]A. O. Barut and H. Kleinert, Phys. Rev. 156, 1541 (1967); 157, 1180 (1967).
[2]B. G. Konopel'chenko, The Group $SO(2,4)+R$ and the Periodic

Table, Preprint, Nuc. Phys. Inst., Siberian Div. USSR Acad. Sci., Novosibirsk, 1972.
[3]I. A. Malkin and V. I. Man'ko, ZhETF Pis. Red. 2, 230 (1965) [JETP Lett. 2, 146 (1965)]. Yad. Fiz. 9, 184 (1969) [Sov. J. Nuc. Phys. 9, 110 (1969)].
[4]Yu. B. Rumer and A. I. Fet, Teoriya unitarnoĭ simmetrii (Theory of Unitary Symmetry), M., 1970.
[5]Yu. B. Rumer and A. I. Fet., Teoret. Mat. Fiz. 9, 203 (1971).
[6]Tsu Yao, J. Math. Phys. 8, 1931 (1967) (I) and 9, 1615 (1968) (II).