CYCLOTRON INSTABILITY OF THE OUTER RADIATION BELT OF THE EARTH UNDER CONDITIONS OF SELF-MODULATION OF GROWING WAVES

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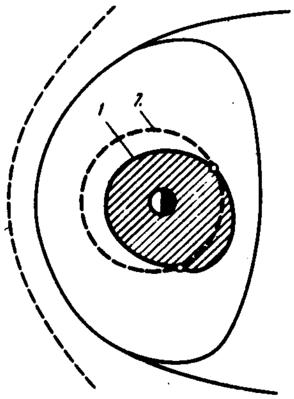
1. The mechanism whereby the earth's outer radiation belt is formed (betatron acceleration [1]) produces an isotropic propagation of the captured particles (particles with large pitch angles predominate). As a result, cyclotron instability develops under favorable conditions. The protons of the radiation belt are unstable against the buildup of Alfven waves ($\omega < \Omega_p$), and the electrons are unstable against waves of the shistler type ($\Omega_p << \omega < \Omega_e$) (cf. [1 - 4]). We shall deal, for concreteness, with Alfven waves [1, 3 - 9].

The Alfven waves propagate along the force lines, becoming reflected from the ionosphere in magnetically conjugate points. Near the equatorial plane



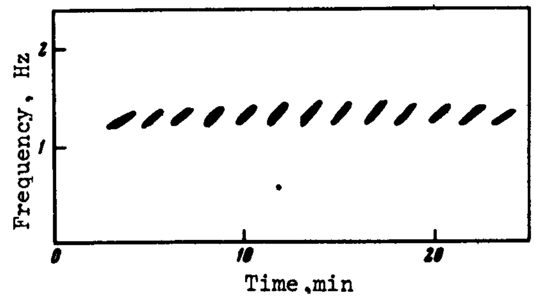


Fig. 1. Equatorial section through the magnetosphere: 1 - plasmopause [10], 2 - drift shell.

Fig. 2. Dynamic spectrum of geomagnetic pulsations of the "pearl" type [3] (schematic representation).

there occurs an effective exchange of energy between the waves and the resonant particles. Instability sets in if the influx of energy from the particles to the waves exceed the loss of the waves in the ionosphere.

It might seem that this should be accompanied by excitation of hydromagnetic noise [7]. Yet in experiment one frequently observes quasimonochromatic signals. It is assumed that such signals propagate in narrow ray tubes. Such tubes are formed, for example, at points where the plasmopause crosses the drift shell of the high-energy particles (Fig. 1).

A remarkable property of quasimonochromatic signals is the fact that they are discrete (Fig. 2). We propose in this paper an explanation of the manner in which discrete signals are produced by continuous excitation.

2. The principal role in the appearance of aperiodic sequence of discrete signals is played by a simple effect (which, judging from the literature, has never been pointed out), namely a rapid displacement of resonant particles across the ray tube, owing to the azimuthal drift in the inhomogeneous magnetic field. If the time that the particle stays inside the tube is shorter than a certain critical value, then self-modulation of the increment takes place and the wave field breaks up into individual packets.

Let the axis of the ray tube be z, and let the x axis be parallel to the proton drift velocity (to the west). We perform an elementary calculation, using the model equation

$$\frac{\partial W}{\partial t} + V_{gr}\frac{\partial W}{\partial z} = (\gamma - \delta)W. \tag{1}$$

Here W is the oscillation energy density, V_{gr} the group velocity, γ the increment, and δ the decrement. The transport equation (1) is based on the assumption that the medium is homogeneous in the z direction. The presence of partly-reflecting boundaries on the two ends of the force line is taken into account by introducing an effective decrement δ and a periodicity condition $W(z + \ell) = W(z)$, where ℓ is the length of the force line.

Assume that the oscillation energy density is homogeneous over the cross section of the tube and is equal to zero outside the tube. Assume that at the eastern edge of the tube ($x = -\Delta x$) the increment is given and equals γ_0 = const. The time evolution of the increment insdie the tube depends on the prior history of the system. We represent the function $\gamma(t)$ phenomenologically in the form $\gamma(t) = \gamma[G(t)]$, where

$$G(t) = \int_{-\infty}^{t} W(t')\Phi(t'-t)\,dt'.$$

We expand γ in powers of G and retain the first two terms:

$$\gamma(t) = \gamma_0 - \alpha G(t). \tag{2}$$

Here $\alpha = -d\gamma/dG$ at $C = 0$. The form factor $\Phi(t' - t)$ is determined on the basis of the following considerations. The replenishment of the particles in the ray tube as a result of a drift with velocity v_{dr} occurs within a time on the order of $\Delta t \simeq \Delta x/v_{dr}$. Therefore $\Phi(\xi) = 0$ when $\xi < -\Delta t$. On the other hand, in the interval $-\Delta t \leq \xi \leq 0$ we put for simplicity $\Phi(\xi) = 1$. Since $G \geq 0$ and $\gamma \leq \gamma_0$, it follows that $\alpha > 0$. The quantity $(\alpha G)^{-1}$ is of the order of time of isotropization of the particles under the influence of the waves.

We now substitute (2) and (1) and obtain

$$\frac{\partial W}{\partial t} + V_{gr}\frac{\partial W}{\partial z} = (\gamma_0 - \delta)W - \alpha W\int_{t-\Delta t}^{t} W\,dt. \tag{3}$$

The equation has a homogeneous stationary solution $\bar{W} = \gamma^*/\alpha\Delta t$, where $\gamma^* = \gamma_0 - \delta$. We shall show that this homogeneous background is unstable against splitting into individual packets.

We put $W = \bar{W} + w$, where $w \sim \exp(iqz - i\kappa t)$, and $\kappa = \kappa' + i\kappa''$. By virtue of the periodicity in z, the quantity q assumes discrete values: $q = 2\pi n/\ell$, $n = 0, 1, 2, \ldots$. Linearizing (3) with respect to w, we get $\kappa' \simeq 2\pi n/\tau$ and $\kappa'' \simeq -\gamma^*(\sin\kappa'\Delta t)/(\kappa'\Delta t)$, where $\tau = \ell/v_{gr}$. We see therefore that the wave envelope amplitudes increase if

$$m - \frac{1}{2} < n\frac{\Delta t}{\tau} < m, \qquad m = 1, 2, \ldots \tag{4}$$

The splitting of the homogeneous background into wave packets proceeds at the fastest rate if $n\Delta t/\tau$ falls in the first interval:

$$\frac{\tau}{2\Delta t} < n < \frac{\tau}{\Delta t}. \tag{5}$$

Let us estimate the number of growing harmonics $n \lesssim \tau/\Delta t$, i.e., the number of packets along the ray tube. The group velocity is close to the Alfven velocity v_a, and the drift velocity is of the order of $v_{dr} \sim v^2/\Omega\ell$, where v and Ω are the velocity and the gyrofrequency of the resonant protons. The half-width of the ray tube is of the order of $\Delta x \sim 1/k$. Taking into account the resonance condition $kv \sim \Omega$, we obtain $\Delta t/\tau \sim v_a/v$. It is known that for pearls $v_a \lesssim v$, so that $n = 1$, as is indeed observed in most experiments [3].

3. Although under real conditions the behavior of the waves and of the particles is more complicated than described above, the main criterion (5) is apparently correct and has a clear physical interpretation.

Assume that there is one priming packet within the length of the tube. With increasing amplitude, a nonlinear regime sets in rapidly, such that the local increment is greatly reduced in one passage of the packet through the interaction region. When $\tau \gtrsim \Delta t$, this prevents the growth of other packets, since the anisotropy in the tube is restored after a time $\sim\Delta t$, i.e., only by the instant the first packet returns. At $\tau \gtrsim 2\Delta t$ it is possible for two packets to appear, etc.

This is the interpretation of the discrete signals excited in the radiation belt. The hypothesis can be directly verified by observing, with a geostationary satellite, the amplitude modulation of the magnetic pulsations (~1 Hz) and the degree of anisotropy of the protons (~10 - 30 keV).

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