RESONANCE IN SUPERCONDUCTORS

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We consider the influence of discrete surface states [1 - 4] on the impedance of a semi-infinite sample $y \geq 0$ in an external magnetic field parallel to the surface, $\vec{H} \parallel z$. The character of such states in superconductors was analyzed in detail in [4]. We choose the natural gauge $A_x = A_x(y)$, $A_y = A_z = 0$, and then the projections of the momenta of the excitations on the x and z axes (P_x and P_z) are conserved, while the energy spectrum $\varepsilon = \varepsilon_n(P_x, P_z)$ has the form shown in Fig. 1 [4]. The magnetic field H_2 at which the gap in the energy spectrum vanishes is $H_2 \sim \Phi_0/\delta\xi_0$, where Φ_0 is the flux quantum, δ is the depth of penetration, and ξ_0 is the coherence length.

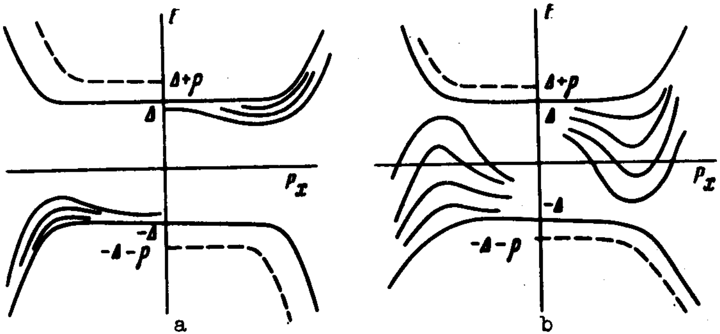


Fig. 1. Form of the energy spectrum in a superconductor: a - $H < H_2$, b - $H > H_2$. The quasidiscrete levels are shown dashed.

The possibility of transitions of the resonant and of the threshold type is obvious already from the form of the energy spectrum, and this leads to corresponding singularities in the surface impedance. Their dependence on the magnitude of the magnetic field, on the temperature, and on the frequency is quite unique. Without stopping to describe the calculations, we present the final result for the resonant increment ΔZ^{res} of the surface impedance:

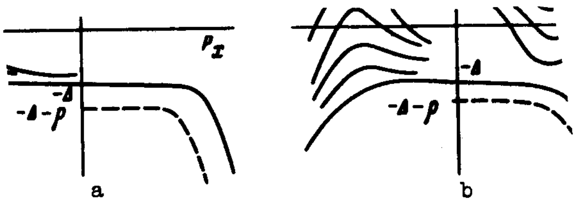
$$\Delta Z^{res} \operatorname{Im} Z \frac{a}{\delta_L}\left(\frac{H}{H_2}\right)^{2/3} \frac{\Delta}{\hbar\omega} \frac{\theta\left(\frac{\omega_{mn} - \omega + \frac{i}{\tau}}{\omega}\right)}{(m-n)^4} [n_F(\epsilon_n) - n_F(\epsilon_m)] . \tag{1}$$

$$\theta(x) = \begin{cases} \dfrac{1}{\sqrt{x}} & \text{for a cylindrical Fermi surface} \\ \ln x & \text{for an arbitrary Fermi surface} \end{cases}$$

$n_F(\varepsilon)$ is the Fermi distribution function. Formula (1) was obtained under the assumption that the resonant frequency $\omega_{mn} = (\varepsilon_m - \varepsilon_n)/h \simeq \omega$, where ω is the frequency of the wave incident on the superconductor; for the same reason as in cyclotron resonance [5], the line broadening and the shift of the resonant frequency as the result of the smearing of the levels can be described by a relaxation time τ. The relative amplitude of the resonance is of the order of a/δ_L for a spherical Fermi surface and of the order of $(a/\delta_L)\sqrt{\omega_{mn}\tau}$ for a cylindrical surface. On the other hand, the derivative of the resonant part of the impedance surface, as usual, is larger by a factor $\omega_{mn}\tau$ than the resonant increment itself.

The line shape in formula (1) determines the dependence of the energy spectrum on one (for a cylindrical Fermi surface) or two (for an arbitrary Fermi surface) continuous parameters P_x and P_z. For the same reason, the frequency of transition from the n-th to the m-th level, which corresponds to the extremal density of states taking part in the resonance, and which is extremal in P_x and P_z, will be resonant. If $m, n \gg 1$ and $m - n \ll m, n$, then the classical orbits corresponding to the levels between which the transition takes place are period in coordinate space, i.e., $\overline{v}_x = \int_0^T v_x dt = 0$, $v_z \doteq 0$. The appreciable dependence on $(m - n)$ in formula (1) makes it possible apparently to observe experimentally only transitions between nearest levels.

The temperature dependence of the amplitude of the resonance is determined by the difference of the occupation numbers of the corresponding levels $[n_F(\varepsilon_n) - n_F(\varepsilon_m)]$. Therefore at $T = 0$ the resonance is possible only if $\varepsilon_m > 0$ and $\varepsilon_n < 0$, corresponding to a transition from a filled level to an empty one. This means that the resonant transitions are possible when there are already discrete levels under the Fermi boundary ($\varepsilon = 0$), i.e., (see Fig. 1), at $H > H_2$. If the temperature differs from zero, then some of the states with energies $\varepsilon_m > 0$ are filled with respect to temperature (and accordingly some of the states with $\varepsilon_n < 0$ are empty), and resonance is realized also at $H < H_2$. The temperature dependence enters not only in the occupation factor but, in accordance with [6], also in the value of the gap, in the depth of penetration, and by the same token in the resonant frequencies. The exponent of H/H_2 depends on the distribution of the magnetic field, but it is always of the order



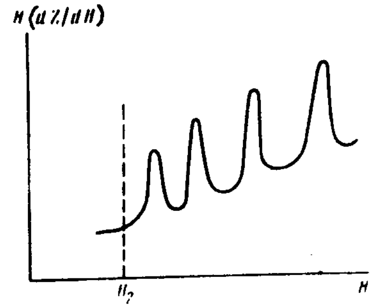


Fig. 2. Phase trajectories corresponding to the following levels: a - discrete, b - quasi-discrete. The level width is determined by the probability of the transition from the state 1 to the state 2, and is exponentially small in the quasiclassical approach.

Fig. 3. Dependence of the derivative of the impedance H(dZ/dH) on the magnetic field. The monotonic part of the curve is only tentative.

of unity, and the presented value 2/3 is obtained in the case of exponential damping of the field, and is more readily an estimate.

Besides the transition between the states of the discrete spectrum, resonance occurs also in transitions from quasidiscrete states to discrete ones (Figs. 1 and 2). Everything stated above can be extended without change also to this case, except that the corresponding resonant frequencies are $\omega^{res} > 2\Delta$.

Another type of singularity is connected with threshold effects, which are due to transitions from states of the discrete spectrum to the continuous spectrum. The picture changes when ω or H changes, and for concreteness we shall consider the change with frequency. With increasing frequency $\omega (h\omega < 2\Delta)$, starting at a certain instant of time, such a transition becomes first allowed, and then the number of the transitions increases periodically with ω with increasing ω. At the same time, the surface impedance also increases. At zero temperature, so long as $H < H_2$, the amplitudes of the increments are exponentially small. The point is that the wave functions that enter in the matrix elements that determines the transition probabilities are not covered in similar fashion by the trajectories of Fig. 2. If $H > H_2$, then the impedance increment is of the same order as the resonance amplitude. At $T \neq 0$, owing to the temperature filling of the levels, just as in the case of resonance, the increments occurring in the magnetic fields $H < H_2$ are likewise not small.

From the character of the energy spectrum it is seen that there are altogether $\sim N(N - 1)/2$ different resonant singularities and $\sim N$ threshold singularities, with H the number of discrete levels ($N \sim (H/H_c)^{1/4}(\delta_L/a)^{1/2}$ if $H < H_2$ and $N \sim \sqrt{(H/H_c)(\delta_L/a)}$ if $H > H_2$.

In conclusion we note that the experimentally observed singularities of the surface impedance in superconductors [7, 8] have apparently a threshold character. To observe the resonance it would be necessary to decrease the frequency of the external field in [7, 8], since the employed frequencies correspond to large m - n in (1).

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