CONTRIBUTION TO THE THEORY OF STRONG INTERACTIONS AT HIGH ENERGIES

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We consider the process of single photon annihilation e^+e^- hadrons. The following is assumed with respect to the decay of a massive quantum with mass M into hadrons. Let $dN_i(p)$ be the number of hardons of type i (pions, K mesons, nucleons, etc.) in an element d^3p of momentun space. Then in the coordinate system where the momentum of the γ quantum is equal to zero there exists a limit for n = 1, 2, ...

$$\lim_{M\to\infty} \langle dN_i(p_1)dN_k(p_2)\cdots dN_1(p_n)\rangle = \frac{d^3p_1 d^3p_2\cdots d^3p_n}{\epsilon_i(p_1)\epsilon_k(p_2)\cdots\epsilon_s(p_n)} f_{ik\cdots s}(p_1, p_2, \dots p_n). \tag{1}$$

In formula (1), $\epsilon_i^2(p) = p^2 + m_i^2$, where m_i is the mass of the hadron of type i.

Hypotheses concerning the existence of limiting properties for the functions describing the result of collisions of high-energy particles were discussed in [1, 2].

The limiting functions $f_{ik\dots s}(\vec{p}_j)$ describe the statistical properties of a certain system. We assume that this system exists and that in different processes at energies W >> m its properties can be measured. It is very likely that in the region of high energies there appear properties of scale independence (similarity) of strong interactions; these properties were proposed in [3, 4]. The similarity properties for a system of functions defined by formula (1) can be readily formulated in analogy with the procedure used in [5] for ordering fluctuations, by introducing fields $\phi_i(p)$ such that we get the correspondence $dN_i(p) \to |\phi_i(\vec{p})|^2 d^3p$ and the mean values in (1) correspond to averaging over a certain ensemble for the fields $\phi_i(\vec{p})$. The following relations are of importance to us:

$$\langle dN_i\rangle = \frac{d^3p}{\epsilon_i(p)^{\alpha}} \chi_i \frac{\epsilon_i(p)}{m}, \tag{2}$$

$$\langle dN_i(p)\,dN_k(q)\rangle - \langle dN_i(p)\rangle\langle dN_k(q)\rangle = \frac{d^3p\,d^3q}{\epsilon_i(p)^{\alpha_i}\epsilon_k(q)^{\alpha_k}} \chi_{ik}\left(\frac{p}{m}, \frac{q}{m}\right), \tag{3}$$

$$\chi_l(y) \approx A_l = \text{const}, \quad y >> 1, \tag{4}$$

$$\chi_{lk}(\lambda, y) \approx \lambda^{-\beta} \chi_{lk}(\lambda x, \lambda y), \quad |x| >> 1, \quad |y| >> 1,$$
$$\chi_{lk}(x, y) \to 0 \text{ for } \left|\frac{x}{y}\right| \to \infty \text{ or } \left|\frac{y}{x}\right| \to \infty. \tag{5}$$

Formulas (2) - (5) are equivalent to the similarity hypothesis, and their experimental verification, as well as a verification of the existence of the limit (1), is critical for the theory. We can present likely arguments, but not rigorous ones, favoring the value $\alpha = 1$.

How are the results of the experiment to be connected in the case of a large but finite value of M with the functions $f_{ik...s}$? Let us consider for concreteness the characteristics of produced pions. We regard the decaying quantum, neglecting the influence of spin, as an elongated object with dimension r_0 and density $\rho(\vec{r})$ (form factor). It is assumed that the result of the decay of this quantum is realized by statistical characteristics of the field $\int\phi(\vec{r}\,')\rho(\vec{r} - \vec{r}\,')dv'$, or in momentum space, of the quantities

$$dN_{\text{eff}} = |\phi(k)|^2 |\rho_k|^2 d^3k. \tag{6}$$

As a result of such an approach, the spectrum is cut off for each sort of hadrons at the momenta p_{0i}, and limiting relations exists for the energies E_i carried away by the hadrons of sort i (see [1])

$$E_l = \lambda_l M, \quad \lambda_l = \text{const for } M \to \infty. \tag{7}$$

We start from the existence of limiting relations for energy (7). The momentum p_0 for the pions we introduce by the relation

$$E_l = \int_0^{p_0} \epsilon(p) < dN(p) > \approx \frac{4\pi A p_0^{4-\alpha}}{4-\alpha}. \tag{8}$$

The multiplicity of the pions is

$$\bar{N} = \int_0^{p_0} < dN(p) > \approx \frac{4\pi A p_0^{3-\alpha}}{3-\alpha} = \text{const } M^{(3-\alpha)/(4-\alpha)}. \tag{9}$$

For the fluctuations of the number of pions we obtain

$$\frac{1}{\bar{N}^2} | < N^2 > - < N >^2 | \approx \text{const } M^{-\beta/(4-\alpha)}. \tag{10}$$

In the calculation of (10) it is assumed that at a given value of p, a contribution is made to the integral with respect to q only by the region of values $p \sim q$. For $\alpha = 1$ we have

$$\bar{N} = \text{const } M^{2/3}, \quad \frac{< N^2 > - \bar{N}^2}{\bar{N}^2} = \text{const } \bar{N}^{-\beta/2}. \tag{11}$$

The construction for other hadrons is analogous.

We now consider the collision between two hadrons a and b. It is assumed that the energy $2W = \sqrt{s}$ of the collision is large. The physical picture of the phenomenon is visualized as follows. In the c.m.s., the hadrons are two narrow disks of radius $\sim 1/m$ and thickness $\sim 1/W$. When these disks collide, a fraction S_{st} of their areas comes in contact. Three systems are produced: "part" a_1 of the hadron a, namely the part of the disk which experiences no collisions, an analogous "part" b_1 of the hadron b, and the part that experiences the collision. A more detailed analysis, which takes into account the fluctuations of the masses and dimensions of the hadrons and a more detailed structure of the colliding part will be given in a separate paper. Here we consider only a rough and qualitative picture. We assume that the disks are always identical and homogeneous, and that the "colliding" part constitutes a system similar to a γ quantum, with a zero c.m.s. momentum. The difference lies in the fact that this quantum is compressed and has a thickness r_0. The "part" a_1 has an energy $W_{a1} \sim W(S_a - S_{st})/S_a$, "part" b_1 an energy $W_{b1} \sim W(S_b - S_{st})/S_b$. Acquiring upon separation a longitudinal momentum $\Delta p_{||} \simeq m^2/W$, the "parts" a_1 and b_1 are transformed into two families of leading particles, carrying away an energy $W_{||d} \sim 2W(1 - S_{st}/S)$. The remaining "quantum" is characterized by a form factor. Within the framework of the picture that leads to the formulas (7) - (11), it is necessary to introduce two limiting momenta for the pions: p_0 and $p_0 \sim m$. For the mean values we obtain the formulas

$$\bar{N} \approx A\pi p_{0\perp}^2 \frac{p_{0\parallel}^{(1-a)}}{1-a}, \qquad (12)$$

$$E_\pi \approx A\pi p_{0\perp}^2 \frac{p_{0\parallel}^{(2-a)}}{2-a} \approx \lambda(2W - W_{||d}).$$

When a = 1 we have

$$\bar{N} \approx A\pi p_{0\perp}^2 \ln\left(\frac{p_{0\parallel}}{p_{0\perp}}\right); \qquad E_\pi \approx A\pi p_{0\perp}^2 p_{0\parallel}. \qquad (13)$$

As shown by experiment [6], α is close to or equal to unity.

The assumptions made here are quite stringent and may not correspond to reality. On the other hand, if they are correct, a very attractive possibility is uncovered of describing the properties of elementary particles in a language of the fields $\phi_i(\vec{p})$ or $\phi_i(\vec{r})$. In the region of low energies $W \lesssim m$, the properties of the fields are complicated, and at $W \gg m$, there is a considerable simplification. Experiments that could be decisive for the verification of these hypotheses seem to be very important.

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