ELECTRONIC THERMAL CONDUCTIVITY IN A TOROIDAL PLASMA LOOP

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A study of the behavior of plasma in Tokamak installations indicates that the plasma loop has an anomalously high thermal conductivity. The heat flow from the plasma in such systems exceeds by more than one order of magnitude the value given by estimates based on the neoclassical theory proposed by Sagdeev and Galeev [1] and further developed in [2, 3]. The heat loss of the plasma loop is not connected with the intensified diffusion, since the average containment time of the plasma particles exceeds by many times the time of conservation of the thermal energy [4]. Measurement of the energy carried away from the plasma by electromagnetic radiation and by fast neutral particles shows that these two factors do not play an appreciable role in the energy balance of the plasma. The energy balance of the ionic component of the plasma is well explained on the basis of the hypothesis that the heat exchange between the electrons and the ions has a classical character and that the ionic conductivity has a classical mechanism [5]. The escape of energy from the plasma along the ion channel turns out to be in this case relatively small. We therefore naturally arrive at the conclusion that the loss of thermal energy in the plasma loop is due principally to the anomalous electronic thermal conductivity. This pertains to ordinary conditions of experiments with the Tokamak installations, in which the plasma is macroscopically stable and the main physical parameters lie between the following limits: average electron concentration $\bar{n}_e$ from 1×10^{13} to 3×10^{13}, average current density in the plasma from 50 to 200 A/cm^2, and longitudinal magnetic field intensity from 20 to 40 kOe.

The coefficient of anomalous thermal conductivity can be estimated by two different methods: 1) on the basis of measurements of the dependence of the thermal energy on the current, 2) on the basis of the measured profile of electron temperature T_e (i.e., the dependence of T_e on r, where r is the distance from the given point in the plasma loop to the annular magnetic axis). The measurement of the reserve of thermal energy in the plasma in a wide range of variation of the physical parameters was carried out with the Tokamak-3a installation. Using the same installation and Thomson scattering of a laser beam, the distribution of T_e over the cross section of the plasma loop was investigated for the first time [6].

It follows from the experimental data that in that phase of the process when the thermal regime in the plasma has already been established, the reserve of thermal energy per unit length of plasma loop W is connected with the current I in the plasma by the relation $W = \gamma I^2$, where $\gamma = 0.35 \pm 0.07$ (W is in ergs and I in cgs emu). For this phase of the process, the energy-balance equation is

$$W/\tau_E = I^2/\pi a^2 \bar{\sigma}, \tag{1}$$

where a is the radius of the cross section of the plasma beam, $\bar{\sigma}$ is the plasma electric conductivity averaged over the cross section, and τ_E is the average time of conservation of energy in the plasma. Taking into consideration the connection between W and I, we obtain for τ_E the expression

$$\tau_E = 0.35 \pi a^2 \bar{\sigma}. \tag{2}$$

Thus, the time of conservation of the energy in the plasma is proportional to the characteristic time of the skin effect for a plasma loop. The ratio τ_E/τ_{skin}, at a steady-state thermal regime, changes little with changing I, T_e, and $\bar{n}_e$. Using (2), we can attempt to find the dependence of the rate of heat transfer in the plasma on the main physical parameters. We shall carry out such an analysis at a crude qualitative level. Let $\bar{\eta}$ denote the average coefficient of temperature conductivity. In the thermal steady state $\tau_E \simeq a^2/6\bar{\eta}$. From this and (2) we get

$$\bar{\eta} \approx \frac{1}{6} \frac{1}{\bar{\sigma}}. \tag{3}$$

The electroconductivity of the plasma is

$$\bar{\sigma} = \frac{\bar{n}_e e^2}{m_e c^2} \frac{1}{\bar{\nu}}, \tag{4}$$

where $\bar{\nu}$ is the generalized frequency of collision of the electrons with ions and with the plasma waves. Consequently

$$\bar{\eta} \approx \frac{1}{6}\frac{m_e c^2 \bar{\nu}}{\bar{n}_e e^2}. \tag{5}$$

The experimentally established connection between W and I can be written also in the form

$$\bar{n}_e k \bar{T}_e (1+\alpha) \approx 1.8\cdot 10^{-2}\overline{H_\phi^2}. \tag{6}$$

Here $\bar{T}_e$ is the average electron temperature per plasma electron and H_ϕ^2 is the square of the intensity of the magnetic field of the current averaged over the cross section of the plasma and calculated from the standard temperature profile (assuming that the current density is distributed in proportion to $T_e^{3/2}$; in this case $\overline{H_\phi^2} = H_\phi^2(a)$). α is the ratio of the average electron and ion temperatures. Substituting in (5) the expression for $\bar{n}_e$ from (6) we obtain, at $\alpha \ll 1$,

$$\bar{\eta} \approx 5\bar{\nu}\overline{\rho_{\phi e}^2} , \tag{7}$$

where $\overline{\rho_{\phi e}^2} = 2m_e k\bar{T}_e c^2/e^2\overline{H_\phi^2}$ is the averaged value of the square of the Larmor radius of the electrons in the field H_ϕ. If a correction is introduced for the ion temperature, then the numerical coefficient in (7) increases to 6 - 7.

It is natural to assume that the local value of η will be expressed by a similar formula with $\bar{\nu}$ and $\overline{\rho_{\phi e}^2}$ replaced by $\nu_{(r)}$ and $\rho_{\phi e}^2(r)$. It is obvious, however, that under the extremely crude estimate and arbitrary averaging, which were used above, we can acquire or lose a numerical factor on the order of 2. We therefore write down the local value of η in the form $\eta = \xi\nu\rho_{\phi e}^2$ and attempt to determine the numerical coefficient ξ in another way, using data on the profile of the function $T_e(r)$. The thermal conductivity equation for the steady state has the following form (at $a/R \ll 1$)

$$\frac{d}{dr}\left(r\kappa\frac{dT_e}{dr}\right) = -qr, \tag{8}$$

where

$$\kappa = \frac{3}{2}n_e k\eta = 3n_e k\xi\nu\frac{m_e kT_e c^2}{e^2 H_\phi^2}, \quad q = \frac{j^2}{\sigma} = j^2\frac{m_e \nu c^2}{n_e e^2},$$

j is the current density. It is assumed here that in the heat balance of the electronic component it is possible to neglect diffusion in the radiation, and also energy transfer from the electrons to the ions.

We note first that the assumed dependence of η on H_ϕ gives a correct description of the function $T_e(r)$ in the "hot zone" of the plasma loop, where this function, according to experimental data, has a "flat top." Indeed, since at $r/a \ll 1$ the quantity H_ϕ^2 is proportional to r^2, we get $dT_e/dr \sim r^3$ and consequently the dependence of the electron temperature on r in the plasma zone near the axis is given by $T_e = T_e(0)(1 - \mu r^4 + \ldots)$, in accord with the experimental results, which are well approximated by the formula and with the plasma waves. Consequently

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$T_e = T_e(0)(1 - r^4/a^4)^2$.[1] Comparing both expressions we find that $\mu = 2a^{-4}$. The connection between μ and ξ can be found by substituting in the thermal-conductivity equation the expressions for κ and q and solving this equation in the region of small values.

We then obtain

$$T_e = T_e(0)\left(1 - \frac{\pi^2}{6\xi}\frac{j_0^4 r^4}{n_e^2(0)k^2T_e^2(0)}\right). \tag{9}$$

Under the previously assumed distribution of the current density over the cross section of the plasma loop, the quantity j_0 (the current density on the axial line) is $\sim 2.2(i/\pi a^2)$. Consequently

$$\mu \approx \frac{4}{\pi^2}\frac{1}{\xi}\frac{I^4}{n_e^2(0)k^2T_e^2(0)a^8}. \tag{10}$$

Measurements of the electron temperature and of the density on the axial line show that the relation $\pi a^2 n_e(0)kT_e(0) \simeq 0.45\ I^2$ is satisfied. Therefore $\mu \simeq (20/\xi)a^{-4}$. We thus find that $\xi \simeq 10$ and consequently

$$\eta \approx 10\nu\rho_{\phi e}^2. \tag{11}$$

The accuracy of this formula depends on the extent to which the empirical formula for $T_e(r)$ approximates the true profile of the electron temperature. We note also that the derivation of the relation between μ and ξ includes a special assumption concerning the dependence of the current density on $T_e(r)$.

The main result of the foregoing estimates can be formulated as follows. On the basis of an analysis of the data on the energy balance of the electronic component of the plasma and the distribution of the electron temperature, we arrive at an expression for the thermal conductivity coefficient outwardly reminescent of the well known classical formula. The difference between them lies in the fact that in place of the intensity of the resultant magnetic field $\sim H_\phi^2$ there appears here a quantity H_ϕ^2, and instead of the Coulomb collisions we have a "generalized" frequency ν, determined from measurements of the thermal conductivity. There is also a coefficient 7 - 10, which increases the heat loss by one more order of magnitude. It is possible, incidentally, that this coefficient is not universal, but depends on the geometric parameters of the plasma loop (on the ratio a/R).

[1] A.A. Galeev and R.Z. Sagdeev, Zh. Eksp. Teor. Fiz. 53, 348 (1967) [Sov. Phys.-JETP 26, 233 (1968)].
[2] L.M. Kovrizhnykh, ibid. 56, 877 (1969) [29, 475 (1969)].
[3] A.A. Galeev, ibid. 59, 1378 (1970) [32, No. 10 (1971)].
[4] L.A. Artsimovich, E.P. Gorbunov, and M.P. Petrov, ZhETF Pis. Red. 12, 89 (1970) [JETP Lett. 12, 62 (1970)].
[5] L.A. Artsimovich, A.V. Glukhov, and M.P. Petrov, ibid. 11, 449 (1970) [11, 304 (1970)].
[6] N.I. Peacock, D.C. Robinson, M.I. Forrest, P.D. Wilcock, and V.V. Sannikov, Nature 224, 488 (1969).

[1] As indicated by the authors of the measurements of the laser scattering.