Weak-interaction neutral currents and the Josephson effect

A. I. Vaĭnshteĭn and I. B. Khriplovich

Institute of Nuclear Physics, Siberian Division, USSR Academy of Sciences

(Submitted May 12, 1974)
ZhETF Pis. Red. **20**, 80–84 (July 20, 1974)

We discuss the possibility of observing parity-nonconserving electron-nucleon interactions with the aid of the Josephson effect.

The observation of weak electron-nucleon interaction would be of great interest to the physics of elementary particles. In this article we wish to call attention to the possibility, in principle, of observing parity-nonconserving eN interactions, at energies on the order of atomic, with the aid of the Josephson effect. The possible manifestations of parity nonconservation in atomic transitions are discussed in[1–3].

The Hamiltonian describing a P-odd contact interaction of two fermions can be represented in first order in v/c in the form

$$\mathcal{H} = \frac{G\hbar^3}{\sqrt{2}c^2}\,\frac{1}{2m}\,[(\beta_1\vec{\sigma}_1 + \beta_2\vec{\sigma}_2)\{\mathbf{p},\delta(\mathbf{r})\}_+ + i\beta_3[\vec{\sigma}_1\times\vec{\sigma}_2]\{\mathbf{p},\delta(\mathbf{r})\}_-]. \quad (1)$$

In this formula, $G = 10^{-5}m_p^{-2}$ is the weak-interaction constant, $\mathbf{r}$ and $\mathbf{p}$ are the relative coordinate and momentum of the fermions, $\vec{\sigma}_{1,2}$ are their spin matrices, β_1, β_2, and β_3 are dimensionless parameters, and the indices plus and minus denote the anticommutator and the commutator. If, e.g., the relativistic Lagrangian of the interaction is represented in the form of a product of currents having a $V-A$ structure, then $\beta_1 = -\beta_2 = \beta_3 = 1$, and m coincides with the reduced fermion mass.

We consider a superconductor with polarized nuclei. To find the effect of the interaction (1) on the motion of the Cooper pairs, we average this interaction over the wave functions of paired electrons with specified summary momentum, and also over the states of the nucleons in the nucleus. The resultant increment to the effective Hamiltonian of the Cooper pairs is

$$\mathcal{H}' = \frac{G\hbar^2 n\beta K}{\sqrt{2}c^2}\,\frac{1}{2m_e}\,\{\mathbf{p},\vec{\zeta}(\mathbf{r})\}_+. \quad (2)$$

Here m_e is the effective mass of the electron, n is the density of the nuclei, $\vec{\zeta}(\mathbf{r})$ is a vector indicating the direction of the nuclear orientation and its modulus is equal to the degree of polarization, while the constant β is expressed in terms of the parameters β_{1p} and β_{1n}, which characterize [see (1)] the interaction of the electron with the proton and with the neutron, by the relation

$$\beta = |\langle \beta_{1p}\sum_p \vec{\sigma}_p + \beta_{1n}\sum_n \vec{\sigma}_n\rangle|. \quad (3)$$

The averaging in (3) is over the state of the nucleons in the nucleus.

The factor K describes the deviation of the current at the nucleus from the average current through the crystal, and is equal to

$$K = \frac{|\,\mathrm{Im}\,\psi_{\mathbf{k}}^*\vec{\nabla}\psi_{\mathbf{k}}|_{\mathbf{r}=\mathbf{r}_{\text{nuc}}}}{|\,\mathrm{Im}\,\psi_{\mathbf{k}}^*\vec{\nabla}\psi_{\mathbf{k}}|_{\text{average}}}, \quad (4)$$

where $\psi_{\mathbf{k}}$ is the wave function of an electron with quasimomentum $\mathbf{k}$, and $\mathbf{r}_{\text{nuc}}$ is the coordinate of the nucleus.

It is easy to verify that, in the presence of an external electromagnetic field described by a vector potential **A**, allowance for the interaction (2) reduces (in first order in G) to the substitution

$$\frac{2e}{c}\mathbf{A}(\mathbf{r}) \to \frac{2e}{c}\mathbf{A}(\mathbf{r}) - \frac{2G\hbar^3 n\beta K}{\sqrt{2}c^2}\vec{\zeta}(\mathbf{r}) \tag{5}$$

in the electromagnetic-interaction Hamiltonian.

Maxwell's equation for a constant magnetic field, with allowance for (5), takes the form

$$\mathrm{rot}\,H = \frac{4\pi}{c}\vec{j} = \frac{4\pi e\rho}{m_e c}\left[\hbar\nabla\phi - \frac{2e}{c}\mathbf{A} + \frac{2G\hbar^3 n\beta K}{\sqrt{2}c^2}\vec{\zeta}(\mathbf{r})\right]. \tag{6}$$

Here ρ is the density of the Cooper pairs and ϕ is the phase of their wave function. Taking the curl of (6), we obtain

$$\left(-\Delta + \frac{8\pi e^2\rho}{m_e c^2}\right) H = \frac{8\pi e\rho G\hbar^3 n\beta K}{\sqrt{2}m_e c^3}\mathrm{rot}\,\vec{\zeta}(\mathbf{r}). \tag{7}$$

Thus, in the presence of polarized nuclei neither the magnetic field nor the current is generally speaking equal to zero in the interior of the superconductor. However, in the usual method of polarization with the aid of an external magnetic field $\mathbf{H}_0(\mathbf{r})$, we obviously have $\vec{\zeta}(\mathbf{r}) \propto \mathbf{H}_0(\mathbf{r})$, so that curl $\vec{\zeta}(\mathbf{r}) = 0$ and neither the magnetic field nor the current penetrates into the interior of the superconductor. We confine ourselves henceforth for simplicity to just this case.

How does the interaction (2) influence the physical effect in superconductors? A change takes place in the condition of quantization of the magnetic flux ϕ through the superconducting loop. It is clear from (5) that this condition now takes the form

$$\frac{2e}{\hbar c}\phi - \frac{2G\hbar^2 n\beta K}{\sqrt{2}c^2}\oint d\mathbf{r}\,\vec{\zeta}(\mathbf{r}) = 2n\pi. \tag{8}$$

By way of another example we consider the flow of current through two Josephson junctions connected and parallel. As is well known, [4] the formula for the maximum current is (we confine ourselves for simplicity to the case of identical junctions)

$$I_{max} = 2I\left|\cos\frac{e\phi}{\hbar c}\right|, \tag{9}$$

where I is the maximum current through one junction, and ϕ is the magnetic flux through the circuit. Allowance for the P-odd interaction (2) leads, just as in the case of the quantization of the flux, to the following modification of (9):

$$I_{max} = 2I\left|\cos\left(\frac{e\phi}{\hbar c} - \frac{G\hbar^2 n\beta K}{\sqrt{2}c^2}\oint d\mathbf{r}\,\vec{\zeta}(\mathbf{r})\right)\right|. \tag{10}$$

Thus, the quantity $I_{\max}$ changes when the sign of ϕ is reversed, i.e., when the relative orientation of the loop and of the external magnetic field is changed.

As seen from (8) and (10), the contribution of the weak interactions is determined by the dimensionless parameter γ,

$$\gamma = \frac{G\hbar^2 n\beta K}{\sqrt{2}c^2}\oint d\mathbf{r}\,\vec{\zeta}(\mathbf{r}) \sim \frac{G\hbar^2 n\beta K|\vec{\zeta}|\,l}{\sqrt{2}c^2}, \tag{11}$$

where l is the length of the closed circuit. It is assumed that the nuclei are polarized along the closed circuit. The constant K can be estimated by using the Wigner-Seitz method[5] to determine $\psi_{\mathbf{k}}(\mathbf{r})$. In the presence of an s or p conduction band this factor is large, $K \sim Z^2\kappa$, where the relativistic correction factor κ also increases with Z and reaches 10 for lead.[2] The increase of the electron wave function near the nucleus is confirmed by experiment on the Knight snift in normal metals. We note that in the case of a pure d-conduction band we have $K \approx 0$. Thus, at $\beta \sim 1$ and $|\vec{\zeta}| \sim 1$ the parameter γ for heavy metals can exceed $10^{-6} l$ cm^{-1}.

We note in conclusion that in principle it is possible to observe in similar fashion P-odd interactions between electrons. To this end it would be necessary to produce in the superconductor polarized electrons, which can apparently be done by placing a thin superconductor with paramagnetic impurities in a magnetic field. At the attainable concentrations of the polarized electrons, however, the effect is quite small.

The authors are sincerely grateful to A.G. Aronov, S.T. Belyaev, V.K. Voĭtovetskiĭ, V.M. Galitskiĭ, V.G. Zelevinskiĭ, A.I. Larkin, V.L. Pokrovskiĭ, R.M. Ryndin, and G.I. Surdutovich for valuable discussions.

[1]Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. **36**, 964 (1959) [Sov. Phys.-JETP **9**, 681 (1959)].
[2]M.A. Bouchiat and C.C. Bouchiat, Phys. Lett. **48B**, 111 (1974).
[3]A.N. Moskalev, ZhETF Pis. Red. **19**, 229 and 394 (1974) [JETP Lett. **19**, 141 and 216 (1974)]; Ya. I. Azimov, A.A. Ansel'm, A.N. Moskalev, and R.M. Ryndin, Yad. Fiz. (in print).
[4]R. P. Reynman *et al.*, Lectures on Physics, Vol. 3, Addison-Wesely, 1965 (Russ. transl., Vol. 9, Mir, 1967, p. 252).
[5]C. Kittel, Introduction to Solid State Physics, Wiley, 1971.