Derivation of an equation of the Biberman-Holstein type for a nonequilibrium radiating diatomic gas

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(Submitted June 3, 1974)
ZhETF Pis. Red. **20**, 95–97 (July 20, 1974)

The problem of the distribution of the density of the vibrational energy of a diatomic gas is reduced to the problem of scattering of completely incoherent radiation.

At decreased pressures and at sufficiently high temperatures of the active degrees of freedom of the molecules, the density of the vibrational energy of a diatomic gas can differ strongly from the equilibrium value, owing to the loss of the vibrational energy by radiation. It is possible to describe nonequilibrium radiation of a diatomic gas on the basis of a pair of kinetic equations, the first of which, describes in the "narrow band" approximation the transport of radiation in a medium with a source function that depends on the populations of the vibrational degrees of freedom, while the second determines the density of the vibrational energy with allowance for the radiation itself[1,2]

$$\mathbf{n}\nabla I_\nu = \kappa_\nu (I_\nu^0 - I_\nu), \tag{1}$$

$$\frac{D\epsilon}{Dt} = \frac{\epsilon^0 - \epsilon}{\tau^0} - \frac{1}{\rho}\nabla\int_0^\infty S_\nu d\nu. \tag{2}$$

Here

$$I_\nu^0 = q\nu^3\epsilon, \quad q = \frac{2m}{c^2\nu_v}. \tag{3}$$

We consider for simplicity an infinite flat layer of a nonequilibrium gas of thickness $2a$ with a fixed pressure and temperature of the active degrees of freedom. Recognizing that at low pressures the individual vibrational-rotation lines do not overlap, we can obtain from (1)–(3) the following equation for the vibrational-energy density

$$\left(\frac{\partial}{\partial t} + \frac{1}{\tau}\right)\epsilon(\xi) = \int_0^1 K(|\xi - \zeta|)d\zeta + \Phi(\xi), \tag{4}$$

where

$$K(\xi) = \frac{\pi q d}{a\rho}\sum_{ja} \tau_{ja}^2 \nu_{ja}^4 \int_{-\infty}^{+\infty}\phi^2(\omega)E_1[\xi\tau_{ja}\phi(\omega)]d\omega, \tag{5}$$

$$\frac{1}{\tau^*} = \frac{2^{5/2}(\pi kT)^{3/2}}{aPm^{1/2}}\left(\frac{\nu_v}{c}\right)^3\sum_{ja}\tau_{ja}\left(\frac{\nu_{ja}}{\nu_v}\right)^4, \quad \frac{1}{\tau} = \frac{1}{\tau^0} + \frac{1}{\tau^*}. \tag{6}$$

Here τ_{ja} is the optical depth across the layer at the center of the line of branch α with rotational quantum number j, $\phi(\omega)$ is the line shape, which can be assumed to be of the Doppler form at low pressures, and $d = (2kT/mc^2)^{1/2}$. The kernel (5) can be represented in the form

$$K(\xi) = \frac{\pi^{3/2} q d \nu_v^4}{a\rho}\int_0^\infty U(x)W\left(\frac{\xi}{x}\right)\frac{dx}{x}, \tag{7}$$

with

$$W(z) = \sum_{ja}\tau_{ja}^2\left(\frac{\nu_{ja}}{\nu_v}\right)^4 e^{-z\tau_{ja}}. \tag{8}$$

We introduce a certain average optical depth σ across the layer at the centers of the lines; we can then write an approximate equation for W:

$$\frac{dW(z)}{dz} = -\sum_{ja}\tau_{ja}^3\left(\frac{\nu_{ja}}{\nu_v}\right)^4 e^{-z\tau_{ja}} \approx -\sigma W(z). \tag{9}$$

To determine σ we use the boundary conditions

$$W(0) = W_1(0), \quad \int_0^\infty W(z)dz = \int_0^\infty W_1(z)dz. \tag{10}$$

Here W_1 is the solution of Eq. (9). Replacing W by W_1 in the kernel (7), we obtain after simple transformations an equation of the Biberman-Holstein type[3] for the density distribution of the vibrational energy

$$\left(\tau\frac{\partial}{\partial t} + 1\right)\epsilon(x) = \frac{R^*}{2}\int_0^\sigma G(|x-y|)\epsilon(y)dy + (1-R^*)\epsilon^0 + \Psi(x), \tag{11}$$

where

$$G(x) = \pi^{-1/2}\int_{-\infty}^{+\infty}\phi^2(\omega)E_1[x\phi(\omega)]d\omega,$$

$$R^* = \frac{1/\tau^*}{1/\tau^* + 1/\tau^0}, \quad \sigma = \frac{\sum_{ja}\tau_{ja}^2\nu_{ja}^4}{\sum_{ja}\tau_{ja}\nu_{ja}^4}.$$

Here $\Psi(x)$ is the source of external radiation. It is easy to prove that R^* has the meaning of the probability of radiative deactivation. We note that R^* exhibits a certain similarity with the albedo of a single scattering. Thus, the problem of nonequilibrium radiation of a diatomic gas in the approximation (9, 10) reduces formally to the problem of scattering of fully incoherent light. This analogy makes it possible, e.g., to write out a general solution of the problem in terms of X and Y functions[4] and by the same token to find the spectral density of the intensity of radiation at any point of space.

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[3] L. M. Biberman, Zh. Eksp. Teor. Fiz. 17, 416 (1947).
[4] V. V. Ivanov, Perenos izlucheniya i spektry nebesnykh tel (Radiation Transport and Spectra of Celestial Bodies), Nauka, 1969.