Theory of resonant ion acceleration by a strong-current relativistic beam

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A theory is constructed for resonant acceleration of ions by a negative-energy wave propagating in a relativistic electron beam.

The recent progress in the production of strong-current relativistic beams and the generation of high-power monochromatic microwave oscillations with their aid have uncovered new possibilities for the realization of the collective method proposed by Faĭnberg for the acceleration of charged particles in a plasma.[1]

One such possibility was considered by Sloan and Drummond[2] and consists in resonantly accelerating the ions by using a negative-energy wave propagating in a magnetized beam. Such a wave can be simultaneously at resonance with an electron beam at the frequency of the anomalous Doppler effect $\omega = k_z v_0 - \omega_{He}$ and at Cerenkov resonance with the ions ($\omega = k_z u$). Ion acceleration by a negative-energy wave should lead to an increase of the wave amplitude, i.e., to instability.[3] The present paper is devoted to the development of a theory of this acceleration method.

In the electrostatic approximation ($\omega \ll k_z c$) the instability produced when the electron-beam space-charge wave interacts with the accelerated ions is described by the dispersion equation

$$\epsilon_e(\omega, \mathbf{k}) = \frac{\omega_{pi}^2}{(\omega - k_z u)^2}\, 0.6(k_\perp a)^2 \frac{k_z^2}{k^2}, \qquad (1)$$

where

$$\epsilon_e(\omega, \mathbf{k}) = 1 - \frac{\omega_{pe}^2}{\gamma^2(\omega - k_z v_0)^2}\frac{k_z^2}{k^2} - \frac{\omega_{pe}^2}{(\omega - k_z v_0)^2 - \omega_{He}^2}\frac{k_\perp^2}{k^2},$$

$$\omega_{He} = \frac{eH_0}{m_e c\gamma}, \quad \omega_{pe}^2 = \frac{4\pi e^2 n_{0e}}{m_e \gamma}, \quad \omega_{pi}^2 = \frac{4\pi e^2 n_{0i}}{m_i};$$

n_{0e} and n_{0i} are respectively the densities of the electron and ion beams, γ is the relativistic factor, and a is the radius of the ion beam; the latter is assumed to move along the axis of the electron beam (in view of the weakness of the transverse fields at the axis, we consider only the longitudinal motion of the ions).

The solution of (1) for the case of a strong magnetic field, $\omega_{He} \gg \omega_{pe}$, is

$$\omega = \left(\omega_{He} + \frac{\omega_{pe}^2}{2\omega_{He}}\frac{k_\perp^2}{k^2}\right)\left(\frac{v_0}{u} - 1\right)^{-1}, \quad k_z = \frac{\omega}{u} + \kappa,$$
$$\kappa = -\frac{1 + i\sqrt{3}}{2^{4/3}}\frac{\omega_{He}}{v_0}\,\delta^{1/3}, \qquad (2)$$

where

$$\delta = 0.6\frac{k_\perp^4}{k^4}\frac{n_{0i} m_e v_0^2 \gamma}{n_{0e} m_i u^2}\frac{\omega_{pe}^4}{\omega_{He}^4}\frac{\omega^2 a^2}{u^2} \ll 1.$$

It follows from (2), in particular, that the phase velocity of the wave excited in the instability exceeds the ion velocity, and the ion flux should become accelerated by the instability. At a constant phase velocity of the wave, this acceleration amounts to $\Delta u \sim u|x|/k_z$, after which the capture of the ions by the wave field prevents them from becoming further accelerated.

To ensure continuous ion acceleration it is necessary, as in ordinary accelerators, to increase the phase velocity in synchronism with the velocity of the accelerated particles. It follows from (2) that at $v_0 \gg u$ this can be done by varying the magnetic field like $H \sim 1/u_s$ (u_s is the velocity of the ions that move in synchronism with the wave). Since the magnetic flux through the beam cross section should be conserved, the density in the beam varies like $n \sim H$ for motion in a decreasing magnetic field.

The nonlinear instability dynamics leading to the ion acceleration can be investigated by a method of partial numerical simulation,[4,5] wherein the electron beam is regarded as a medium with specified dielectric constant $\epsilon_e(\omega, \mathbf{k})$, and the motion of the resonant ions in the wave field is essentially nonlinear.

In this approximation, the electric potential of the accelerating wave is expressed in the form

$$\phi(t, \mathbf{r}) = \phi(z) I_0(k_\perp r)\cos\left(\int_0^z k_z dz - \omega t + a(z)\right).$$

The system of equations for the variation of the amplitude and phase $\varphi(z)$ and $\alpha(z)$ of the field and the ion velocity $u(z)$, when expressed in terms of the dimensionless variables

$$\zeta = \kappa_0 z, \quad \psi = \frac{e\phi\omega^2}{m_i \kappa_0^2 u^4(0)}, \quad \tau = \frac{\omega}{2\pi}\left(t - \frac{z}{u(0)}\right), \qquad (3)$$

$$\frac{d\tau}{d\zeta} = -\frac{u - u(0)}{\nu u}, \quad \kappa_0 = \delta^{1/3}(0)\frac{\omega_H(0)}{v_0} \qquad (4)$$

takes the form

$$\frac{d\psi}{d\zeta} = C(\zeta)\int_{-1/2}^{1/2}\left(1 + \nu\frac{d\tau}{d\zeta}\right)\sin\Phi\, d\tau_0,$$

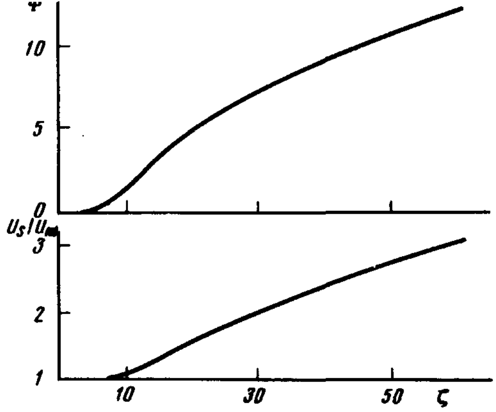

FIG. 1. Dependence of the ion velocity and of the potential amplitude on the distance under conditions of synchronous interaction.

$$\psi \frac{da}{d\zeta} = C(\zeta) \int_{-1/2}^{1/2} \left(1 + \nu \frac{dr}{d\zeta}\right) \cos \Phi \, dr_0 ,$$

$$\frac{d^2 r}{d\zeta^2} = -\frac{1}{2\pi} \left(1 + \nu \frac{dr}{d\zeta}\right)^3 \left(1 + \frac{\nu}{2\pi} \lambda(\zeta)\right) \psi \sin \Phi , \qquad (5)$$

$$\int^{\zeta} \lambda(\zeta)\, d\zeta = \Phi_s + 2\pi r_s - a .$$

Here $\mu = k_\perp^2(0)/k_z^2(0)$ and $\nu = 2\pi x_0/k_z(0)$ are parameters of the problem, $C(\xi) = [(1+\mu)/(1+\mu+\nu\lambda(\xi)/2\pi)]^2$, $\Phi = \alpha - 2\pi\tau + \int\lambda d\xi$, and $\lambda = [k_2(z) - k_z(0)]/x_0$. The variation of the wave number along the system is determined from the synchronism condition [the last equation of (5)] – the phase of the field acting on the synchronous particle remains constant at Φ_s used in the integration of the equations in (5) was chosen to satisfy the condition that the maximal number of ions be captured in the synchronous motion.

The results of the numerical integration of Eqs. (5) are given in Figs. 1 and 2. At $\psi \ll 1$ the field amplitude has an exponential increase with ξ, in accord with the linear theory. In the case of a constant phase velocity of the wave stabilization of the instability should occur at $\psi \sim 1$. In the presence of synchronism, the amplitude continues to increase monotonically with ξ, and at sufficiently large ξ both ψ and u_s are proportional to $\sqrt{\xi}$. On the phase plane (see Fig. 2), the instability leads to a breakup of the ion beam into particle bunches that oscillate relative to the synchronous phase $\Phi_s \approx 0.8\pi$[1)] and are accelerated by the wavefield. Up to 80% of the beam particles are then set to move in synchronism. As is usual in linear accelerators, the acceleration of a synchronous particle is accompanied by damping of the phase oscillations and by contraction of the bunch. Under these conditions, the single-mode regime produced as usual by x preliminary modulation of the electron beam turns out to be stable, since no satellite instability[7] develops and the only amplitude that increases is that of the wave for which the synchronism condition is satisfied.[2)]

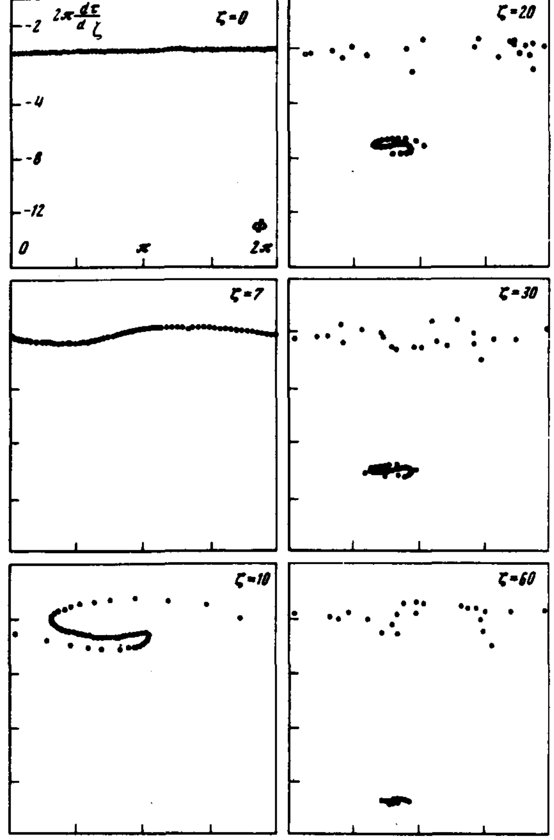

FIG. 2. Phase plane of ion beam.

At sufficiently large ξ, when most ions are captured into synchronous bunches, the law governing their acceleration can be obtained by integrating the simple equations that describe the synchronous ion motion and the conservation of the energy flux in the ion beam + wave system in a medium with a dielectric constant $\epsilon_e(\omega, \mathbf{k})$:

$$\frac{k_\perp^2 \phi^2}{8\pi} - 0.3 n_{0i} m_i u_s^2 \frac{\omega_{pe}^2(0)}{\omega_H^2(0)} k_\perp^2 a^2 = \text{const} , \qquad (6)$$

$$m_i u_s \frac{du_s}{dz} \quad \frac{e\omega}{u_s} \phi \sin \Phi_s . \qquad (7)$$

It follows therefore that at $u_s < c$ the ion energy increases like

$$\frac{u_s^2}{u^2(0)} = 0.7 \frac{k^2(0)}{k_\perp^2(0)} \sin \Phi_s \, \eta^{1/2} \delta^{1/2}(0) \frac{\omega_H(0)}{v_0} z . \qquad (8)$$

Here η is the ion capture coefficient. Stabilization sets in and the acceleration stops because the electron beam goes out of synchronism with the wave at a deceleration $\Delta v_z \sim -\omega_{pe}^2/\omega_H k_z$. The maximum power transferred to the ion component is given by

$$
W_i = \begin{cases} n_0 e m_e c^3 \gamma \times 2\gamma^2 \dfrac{\omega \omega_{pe}^2}{\omega_H^3}, & \text{if } \dfrac{\omega_{pe}\gamma}{\omega_H} < 1 \\ n_0 e m_e c^3 \gamma \times \dfrac{2\omega}{\omega_H} & \text{if } \dfrac{\omega_{pe}\gamma}{\omega_H} > 1 \end{cases}, \qquad (9)
$$

It follows from (8) and (9) that using an electron beam with parameters $I_e = 10^5$ A and $E_e = 3\times10^6$ eV and with a pulse duration $\tau \approx 10^{-6}$ sec it is possible to develop a proton accelerator with an output energy $E_p = 3\times10^8$ eV and with $N_p \approx 2\times10^{15}$ particles per pulse, at an accerator length $L \approx 5$ m over which the magnetic field varies from 10^5 to 3×10^3 Oe. We note that this method can accelerate protons and multiply-charge ions with equal efficiency.

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1) At the chosen form of the field, the accelerated particles have phase and radial stability at $\pi/2 < \Phi < \pi$.

2) The condition for synchronism with the ions is likewise not satisfied for the Cerenkov harmonic of the space-charge wave $\omega \approx k_g v_0 - \omega_{pe}/\gamma$, so that this wave does not affect significantly the acceleration process.

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