Certain singularities of the thermocirculation effect in superconductors and in a superfluid liquid

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Attention is called to the possible occurrence of hysteresis and other phenomena in unevenly heated superconducting circuits and in annular vessels containing superfluid liquid. These phenomena can be used to measure the circulation quantum.

As is well known, a magnetic field should be produced in an inhomogeneous superconductor in the presence of a temperature gradient, together with a current that flows around the inhomogeneity.[1,2][1)] In the particular case of an inhomogeneous closed circuit consisting of two different superconductors, the temperature gradient should produce a circulating current.[3,4][2)] The existence of such an effect was observed experimentally by Zavaritskiĭ.[5] This effect turns out to be very small, but it can be amplified.

Consider a planar closed superconducting circuit (heat cell) consisting of several bulky links whose ends are maintained at different temperatures (Fig. 1). We take a closed loop $\mathcal{L}$ that is entirely contained in the interior of the superconducting circuit. It is obvious that the current in this loop is zero

$$\mathbf{j} = \mathbf{j}_s + \mathbf{j}_n = 0, \tag{1}$$

where $\mathbf{j}_s$ is the usual superconducting current and $\mathbf{j}_n = b\nabla T$ is the thermal current. Integrating (1) along the indicated loop, we obtain, in analogy with[2], the magnetic flux linked with the loop

$$\Phi = \Phi_T + \Phi^{(0)} \tag{2}$$

$$\Phi_T = m\frac{4\pi}{c}\int_{T_1}^{T_2}[(b\delta^2)_{\mathrm{I}} - (b\delta^2)_{\mathrm{II}}]dT, \quad \Phi^{(0)} = n\Phi_0, \ \Phi_0 = \frac{hc}{2e}. \tag{3}$$

Here m is the number of links of the thermal cell made up of superconductors I and II, δ is the London penetration depth, and n is the number of flux quanta enclosed by the loop $\mathcal{L}$ at $T_1 = T_2$.

It is seen from (3) that Φ_T can become arbitrarily large in a circuit consisting of a sufficiently large number of links.[3)] The flux in such a thermal cell should therefore increase smoothly[4)] when the temperature difference $\Delta T = T_2 - T_1$ is increased gradually from $\Phi = 0$ (if no flux was initially linked by the circuit, i.e., $n = 0$) to $\Phi \lesssim \Phi_c$, where $\Phi_c = H_c^* S$, S is the area of the loop (Fig. 1), and $H_c^* = \min\{H_{c\mathrm{I}}, H_{c\mathrm{II}}\}$ is the critical field of a circuit of type-I superconductors. Obviously the total flux cannot exceed the value $\Phi \approx \Phi_c$ at which the weak section of the circuit would be destroyed. Therefore a flux $\Phi^{(0)}$ of polarity opposite that of Φ_T appears spontaneously in the circuit and the terms Φ_T and $\Phi^{(0)}$ cancel out in (2).

Since $\Phi^{(0)}$ is quantized, jumps should appear on the $\Phi(\Delta T)$ curve at $\Phi \approx \Phi_c$ (Fig. 2), and the curve should take a sawtooth form with tooth amplitude proportional to the number of produced flux quanta. The appearance of these quanta could be revealed also by the hysteresis of $\Phi(\Delta T)$ in this region, namely, a frozen-in flux $\Phi = \Phi^{(0)}$ should remain in the circuit as $\Delta T \to 0$, even though this flux was not present initially (see curves 1

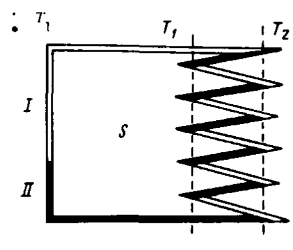

FIG. 1.

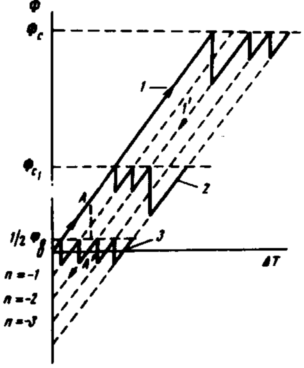

FIG. 2.

$-1'$ in Fig. 2). If type-II superconductor sections are connected in the circuit of Fig. 1, then the transition to the sawtooth regime takes place at $\Phi \lesssim \Phi_{c1}$, where $\Phi_{c1} = H_{c1}S$ and H_{c1} is the lower critical field at which vortices begin to be produced in the type-II superconductor (curve 2 of Fig. 2).

We note that from the point of view of thermodynamics, a superconducting ring with frozen-in magnetic flux is metastable, since the state without the flux is more convenient. It is therefore possible that the transition to the sawtooth regime occurs in fact not when the thermodynamic value Φ_c is reached, or Φ_{c1}, but sooner. Under equilibrium conditions (in the absence of superheating and supercooling),[6] the sawtooth regime would be observable already at a flux $\Phi \approx \Phi_0/2$ (curve 3 of Fig. 2).[5)]

If a section with weak coupling (a Josephson junction) is connected in the circuit, a regime similar to curve 3 of Fig. 2 should also be observed. For the Josephson junction, however, the function $\Phi(\Delta T)$ should have a smooth sinusoidal shape rather than a sawtooth shape.

We note finally the following hysteresis effect. The metastable state of the superconducting loop, represented by point A on Fig. 2, is reversible: $\Phi \to 0$ as $\Delta T \to 0$. However, if the circuit is opened and again closed at the point A, then the loop goes over to an equilibrium state A' corresponding to the minimal Φ. If we now let $\Delta T \to 0$, a frozen-in flux remains in the loop.

Similar phenomena should be observed also in the thermomechanical circulation effect in a superfluid liquid. Then $\Phi^{(0)}$ in Fig. 2 should be taken to mean the velocity of the reactive rotation of the freely suspended ring $V_R = n^{-1} M_n^{-1} \int \rho_s v_s^{(0)}\, dV = M_n^{-1} \int J_s^{(0)}\, dl = J_s^{(0)} L/M_n$, where $J_s^{(0)}$ is the total superfluid current circulating in the ring, L is the length and M_n is the normal mass of the ring (including the masses of the walls). The characteristic values of V_R corresponding to one circulation quantum

$$V_R(n=1) = \frac{2\pi\hbar}{m_{He}L}(1+\xi)^{-1}\frac{M_s}{M_n} \approx 10^{-3}\frac{M_s}{M_n}(1+\xi)^{-1} \text{ cm/sec},$$

at small values of M_n/M_s and $\xi = (l_I/S_I + l_{II}/S_{II})S/L$ are perfectly observable (l_i and S_i are the lengths and cross section areas of the capillaries).

If desired, the thermomechanical circulation effect in a superfluid liquid can be enhanced by connecting in series several thermal links. Actually, however, in the case of helium II, there appears to be no need for this procedure. Indeed, the temperature difference needed to produce at the ends of one link (capillary) a phase difference $\Delta\phi_T \sim 2\pi$ is equal to[2]

$$\Delta T = \frac{\rho_s}{\rho_n}\frac{8\eta_n l v_s T}{\sigma\rho r^2} = \frac{\rho_s}{\rho_n}\frac{8\eta_n}{\sigma\rho r^2}\frac{2\pi\hbar}{m_{He}} \sim 10^{-2}\ {}^\circ\text{K}$$

at $T \sim 1\,{}^\circ$K and a capillary radius $r \sim 10\ \mu$. With increasing T and (or) with increasing r, the value of ΔT decreases rapidly.

When the temperature gradient in the ring with He II is gradually increased, hysteresis effects and spontaneous transitions, analogous to those shown in Fig. 2, should be observed. The frequency of these transitions is determined by the rate of formation of the vortex nuclei, and when the velocity $v_s^{(0)}$ on any section of the ring approaches the critical vortex-formation velocity this frequency should increase strongly (this corresponds to the transition to the sawtooth regime in Fig. 2). If the critical value is reached by the velocity v_s^T in the thermal flow before it is reached by $v_s^{(0)}$, then the ring goes over into a regime of small random beats (curve 3 in Fig. 2). Observation of jumps of the velocity V_R, and observation of residual circulation flows that remain in the ring after such jumps when the temperature gradient is eliminated, make it possible, in principle, to measure the value of the circulation quantum in He II. The described phenomena are also of independent interest, being one more manifestation of macroscopic quantum effects in superconductors and in He II.

The authors thank V. L. Ginzburg for a discussion of the problems touched upon in the paper.

1)An effect of the same type is produced in unevenly heated anisotropic superconductors.[1] In this paper we consider isotropic superconductors.

2)The effects discussed in[1] and in[3,4] are physically equivalent and are due precisely to the inhomogeneity of the superconductor. This circumstance was noted in[2].

3)The possibility of amplifying the thermoelectric effect in a circuit of this type was noted in[2]. It the area S of Fig. 1 is filled with a superconducting core, then the magnetic field, and hence also the current in the loop $\mathcal{L}$, is additionally amplified.

4)It is curious that the flux Φ assumes in this case arbitrary values, i.e., it is not quantized. This is due to the presence of the term j_n in expression (1) for the total current. Of course, the usual theorem concerning flux quantization holds true in the case of a purely superconducting current $j = j_s$.

5)Curves 1—3 in Fig. 2 are made up of linear sections. If the temperature dependence of the coefficient $b\delta^2$ in (3) is taken into account, the sloping sections become curvilinear.[3,5]

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