Development of instability and loss of symmetry following isentropic compression of a spherical drop

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We consider the dynamics of isentropic compression of a drop with an inhomogeneous initial density distribution under the influence of an external pressure that is constant over the surface of the drop. It is shown that the spherical drop shape is unstable and that the spherical symmetry is lost in the course of compression.

In connection with the problem of laser-induced thermonuclear fusion, many authors have calculated the compression and heating of spherical targets that are symmetrically irradiated by laser pulses of specially programmed form. To study the compression dynamics and to choose the optimal laser-pulse waveform, use was made of both numerical methods[1–4] and of an analytic approach based on particular solutions of the gas dynamics equations.[4–6] In either case, only motions having spherical symmetry were investigated with a sufficient degree of thoroughness. Simple qualitative considerations indicate that such motions are unstable; so far, however, no systematic investigations were made of laser compression with allowance for the instabilities that disturb the spherical symmetry.

We have investigated the nonlinear development of the perturbations following the compression of a drop by an external pressure applied to its surface, i.e., we disregarded the external "corona," which was replaced by the pressure field. The instabilities of the corona can only make matters worse. This compression process seemed to be a reasonable model for the description of the behavior of a dense core of a laser target during the course of the irradiation. A spherically symmetrical variant of this model was considered in connection with problems of laser-induced thermonuclear fusion in[4,5]. We consider the more general case of drops with ellipsoidal shapes, and use the results of[7], where the problem was solved of the expansion, in vacuum, of a gas cloud having no spherical symmetry. The motion of interest to us can be obtained as a result of the time reversal of the flow considered in[7]. This time-reversed flow is unstable, and the limiting shape of the drop differs from spherical more the higher the degree of compression. The solution obtained below is exact. It assumes neither slowness of the process nor constancy of the pressure in the drop.

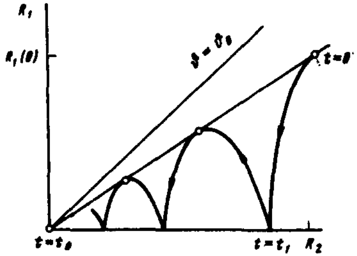
Proceeding to solution, we consider the class of motions of a compressible medium described by an affine coordinate transformation $r_i = F_{ik}(t)a_k$ (r_i are the Euler coordinates and a_k the Lagrangian coordinates of the liquid particle; $i, k = 1,2,3$). This class includes deformation sand rotations of an arbitrary triaxial ellipsoid.[8,9] The motions considered in[4,5] also belong to this class and reduce to a simple scale transformation $r = aF$. To demonstrate the instability of such symmetrical motions, it suffices to consider pure deformation in the absence of rotation. In this case the transformation matrix is diagonal, $F_{ik} = R_i\delta_{ik}$. The system of gas dynamics equations for such flows can be reduced to a system of ordinary differential equations for the components of the vector $R_i(t)$

$$\ddot{R}_i + \frac{\partial U}{\partial R_i} = 0; \quad U = -c(R_1R_2R_3)^{-2/3}; \quad c > 0 \tag{1}$$

[in the derivation of (1) it was assumed that the compressed matter has a power-law adiabat with exponent $\gamma = 5/3$].

In classical mechanics, the system (1) describes the falling of a particle on a center in an asymmetrical potential field. An analysis of the solution of the system (1) becomes easier if one uses the energy integral and the additional integral obtained in[7]:

$$R_i R_i = 2Et^2 + At + B.$$



(A, B, and E are integration constants). An investigation shows that the fastest compression takes place along the axis corresponding to the smallest initial radius $R_i(0)$ (we shall designate this axis by the index 1). After a finite time t, the ellipsoid "collapses" completely, i.e., $R_1(t_1)$ vanishes while R_2 and R_3 remain finite. The velocity component along the axis 1 increases without limit as $t \to t_1$. The solution can be formally continued beyond the collapse point, if we consider the reflection from the plane $R_1 = 0$. The figure shows the solution of the problem in the case of compression of a spheroid ($R_2 = R_3$, $\theta = \tan^{-1}[R_1/R_2\sqrt{2}]$). The value $\theta_0 = \sin^{-1}(1/\sqrt{3})$ corresponds to spherically-symmetrical compression. Let the initial deviation of the drop shape from spherical be small, $|\theta(0) - \theta_0| = \Delta_0 \ll 1$. Simple calculation shows that upon compression the deviation from sphericity increases like

$$\Delta = \Delta_0 \left(1 - \frac{t}{t_0}\right)^{-\alpha}, \qquad (2)$$

where t_0 is the positive root of the expression $2Et^2 + At + B$ and $\alpha = 1/2$. Introducing the average degree of compression n, equal to the ratio of the initial volume of the drop to the final one, we can rewrite (2) in the form

$$\Delta = \Delta_0 n^{\beta}; \quad \beta = \frac{\sqrt{2}}{3} \approx 0.5 .$$

Thus, isentropic compression increases the deviation from spherical shape by a factor $\sqrt{n}$.

We note that the obtained solution corresponds in essence to the nonlinear stage of development of a Taylor instability in a compressible medium for a particular form of the initial perturbation, with a wavelength on the order of the radius of the compressible drop. It is understandable that the amplitude of such a perturbation should increase appreciably within a time on the order of the total compression time t_0. For perturbation with a smaller wavelength the growth time decreases in proportion to $\sqrt{\lambda}$. It is natural to expect, however, that the fastest growing short-wave perturbations in the considered situation will be suppressed as a result of the smearing of the boundary between the dense core and the corona, so that the actual character of the deviation from spherical symmetry upon compression will correspond in general outlines to the considered model.

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