# Scalar-tensor theory of gravitation

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It is noted that the hypothesis previously advanced by the author, that the Lagrangian of the gravitational field is zero, leads in principle to the impossibility of observing the scalar field of the scalar-tensor gravitation theory, which thus goes over into Einstein's pure-tensor theory. This conclusion is the result of the special form of the dependence of the gravitational constant on the scalar field, $G=\mu^{-2}$ ($\mu$ is the scalar field, and the particle masses are proportional to $\mu$). It is also shown that at any other dependence of $G$ on $\mu$, e.g., at $G=$const, the scalar-tensor theory leads to violations of the principle of equivalence of the inertial and gravitational masses.

The scalar-tensor theory of gravitation and its possible experimental consequences have been under discussion in the literature for a number of years.[1] Attention is called in this article to important consequences that follow for this theory from the hypothesis of the zero gravitational-field Lagrangian (ZL), advanced by the author in 1967.[2] It follows from the ZL hypothesis that the scalar field is unobservable and is excluded from the theory, which thus goes over into Einstein's usual pure-tensor theory.

If, however, we forgo the ZL hypothesis, then the scalar field becomes manifest in observable effects. But at the same time, it is revealed that it is impossible to satisfy the condition of equivalence (proportionality) of the interial and gravitating masses. We do not regard a theory with the equivalence principle violated as satisfactory.

In the scalar-tensor theory of gravitation one introduces, besides the tensor gravitational field $g_{ik}(x)$, also a certain scalar field $\mu(x)$. It is assumed that the masses of all bodies depend on the coordinates and the time and are proportional to a common variable scale field $\mu(x)$. We shall show below that in the general case this condition cannot be satisfied in a noncontradictory manner, but for the time being we confine ourselves to the traditional treatment, which is actually valid only if the gravitational mass defect is neglected. The action for a material point is equal to

$$\left.\begin{array}{l} S_t = -\int m(x)\sqrt{dx^i\,dx^k g_{ik}} \\ m(x) \sim \mu(x) \end{array}\right\}\ldots \tag{1}$$

For a classical trajectory we have $\delta S_t=0$. The trajectory of a point is thus determined by the fields $g_{ik}$ and $\mu$.

The action (1) is invariant against the scale transformation

$$\left.\begin{array}{l} g^{\circ}_{ik} = \mu^2 g_{ik} \\ m^{\circ} = m/\mu = \text{const} \\ dS^{\circ} = \mu dS \end{array}\right\} . \ldots\ldots\ldots\ldots\ldots\ldots \tag{2}$$

Here $dS$ is an interval in a gauge that admits of the variable $\mu$ field, and $dS^0$ is the transformed interval.

The transformation (2) was investigated in great detail by Dicke.[1]

It turns out that it is just the interval $dS^0$ which has a direct physical meaning. Let us consider by way of example the measurement of time by an atomic clock that moves along a world line in a field $\mu(x)$. The frequency of the atomic oscillations is $\omega=\Delta E/\hbar$ ($\Delta E$ is the difference between the atomic levels). The energy is proportional to the rest mass, i.e., to $\mu$. The frequency $\omega$ is proportional to $\mu$. The time interval measured by the atomic clock is therefore $\mu\,dS=dS^0$. This conclusion can be directly extended to include clocks that use other nongravitational phenomena (elastic oscillations, the earth's diurnal rotation, etc). But the extension of this conclusion to gravitational effects calls for the use of additional information on the dependence of $\mu$ on the gravitational constant $G$. For example, the oscillation frequency of a pendulum is $\omega=\sqrt{GM/R^2 l} \sim G^{1/2}\mu^2$ ($M$ is the mass of the earth and is proportional to $\mu$, $R$ is the radius of the earth and is proportional to $\mu^{-1}$, and $l$ is the pendulum length and is proportional to $\mu$). If $G\sim\mu^{-2}$, then all clocks vary in the field $\mu$ in the same manner, $\omega\sim\mu$, and the field $\mu$ is in fact unobservable and is excluded from the theory by the scale transformation (2). The physical consequences of such a theory agree fully with Einstein's general relatively theory (at least in the classical theory).

If we forgo the condition $G\sim\mu^{-2}$, then the field $\mu$ is not eliminated by the transformation (2) and becomes manifest in a number of phenomena (the pendulum oscillation frequency is not proportional to $\mu$, $G^0=\mu^2 G$ is not a constant, etc.). At the same time, however, the equivalence principle is violated. For example if we take the assumption made in[1] that $G=$const and $G^0\approx\mu^2$, then the gravitational mass defect is proportional not to $\mu$ but to $\mu^3$. Such a theory seems very unlikely.

The relation $G\sim\mu^{-2}$ and the possibility of eliminating the scaler field by a scale transformation follows from the zero-Lagrangian hypothesis.[2] It is assumed that in a consistent quantum field theory the Lagrangian function and the total action reduce to the Lagrangian function of matter $L_m$ and to the action of matter $S_m=\int\sqrt{-g}L_m(dx)$, while the action of the gravitational field (phenomenologically equal to $S_g=\int\sqrt{-g}(R/16\pi G)(dx)$) is a polarization effect. The action of matter $S_m$ is invariant to the transformation (2) at $L_m\sim\mu^4$. The ZL hypothesis extends this invariance to include gravitational phenomena.

*Conclusion*. A scalar-tensor gravitation theory whose conclusions differ from those of general relativity theory cannot be reconciled with the hypothesis that the gravitational field has a zero Lagrangian and with the equivalence principle.

[1]P. Jordan, Astr. Nach. 276, 1955 (1948); Scwerkraft, Weltall, 1959; G.R. Tirry, Compt. rend. **226**, 216 (1948); P. Bergmann, Ann. Math. **29**, 255 (1948); C. Brans and R.H. Dicke, Phys. Rev. **124**, 925 (1961); R.H. Dicke, Phys. Rev. **125**, 2163 (1962).

[2]A.D. Sakharov, Dokl. Akad. Nauk SSSR **177**, 70 (1967) [Sov. Phys.-Dokl. **12**, 1040 (1968)] A.D. Sakharov, Article 3 in Preprint Collection of Applied Mathematics Institute "Gravitation and Field Theory," Oct., 1967; Paper at Seminar, Phys. Inst. Acad. Sci., June 1970.

# Erratum: Scalar-tensor theory of gravitation [JETP Lett. 20, No. 3, 81–82 (August 5, 1974)]

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The abstract contains the phase "It is also shown that for any other dependence of $G$ on $\mu$, e.g., at $G = \text{const}$, the scalar-tensor theory leads to violations of the principle of equivalence of the inertial and gravitational masses." This is an inappropriate phrase and should be eliminated from the text. Violations of the equivalence principle were known to all authors dealing with scalar-tensor theory, and were at the center of their interest. On page 81, in line 18 of the right-hand column it is necessary to make the following correction: the length of the pendulum is $\mu^{-1}$. In line 23 it is necessary to omit the words in the parentheses. In the last line of the article it is necessary to eliminate the words "and with the equivalence principle." (Communicated by A.D. Sakharov on October 30, 1974).

  