Estimate of cross sections on nuclei at extremely high energies in a model with a logarithmically increasing scattering phase shift

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It is shown that the nucleon-nucleon interaction model in which the scattering phase shift increases logarithmically with increasing energy can be reconciled both with the data on proton-proton interactions at ISR energies, and with the data on the cross sections of protons on nuclei at ultrahigh energies.

In[1] I proposed an eikonal model of nucleon-nucleon scattering at high energies, in which the scattering phase shift increases logarithmically with increasing energy. It was noted in[2] that this model can describe the ISR data,[3] according to which the *pp*-scattering cross section increases like the square of the logarithm of the energy in the region of 10^3 GeV. At the same data, cosmic-ray data are available on the cross sections for the interaction of nucleons with air-atom nuclei, up to energies on the order of 10^8 GeV.[4-7] These data are rather approximate, but they indicate that the cross sections increase by less than a factor 1.5 up to such

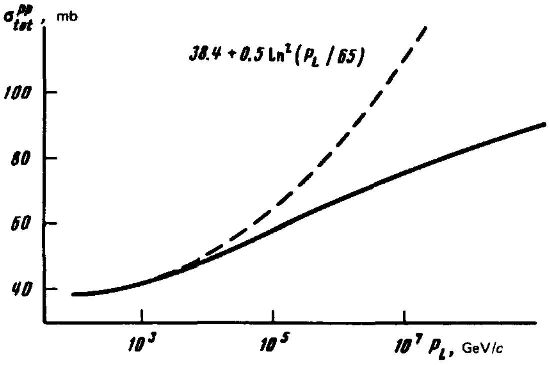

FIG. 1. Total nucleon-nucleon interaction cross section.

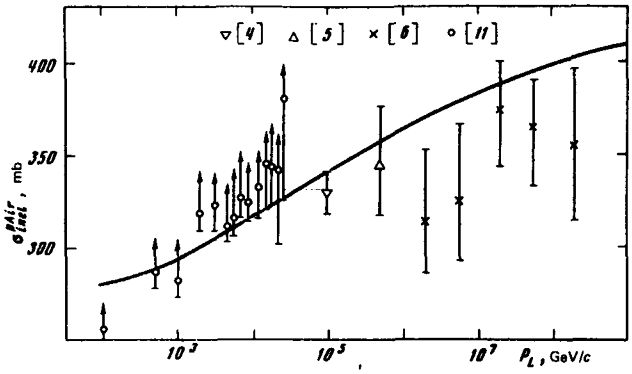

FIG. 2. Inelastic cross section on air-atom nuclei (the curve was calculated for ^{14}N nuclei).

energies. This paper presents an estimate of the inelastic nucleon-nucleus cross section within the framework of the model of[1,2]. It will be shown that this model can also describe the relatively weak growth of the cross section on nuclei at cosmic-ray energies.

According to the model of[1,2], the growth of the cross section at high energies is ascribed to a new process—exchange of neutral vector mesons that interact with one another. The scattering phase shift is represented in this case as a sum of two parts: $\overline{\chi}$, corresponding to a constant cross section, and $\Delta\chi$, corresponding to a new process. The latter contribution to the phase shift is determined by the characteristics of the meson-meson interaction and if these characteristics are approximately constant, it increases with energy like $\ln^2 p_L$, where p_L is the momentum in the laboratory frame. The coefficient of the term $\ln^2 p_L$ turns out to be small, so that at ISR energies the increment to the cross section is proportional to $\Delta\chi$ and takes the form $\Delta\sigma \ln^2 p_L$. With further increase of the energy, the growth of the cross section should slow down, leading formally to proportionality to $\ln\ln p_L$ as $p_L \to \infty$. Generally speaking, the meson-meson cross section itself can also increase with increasing energy. This, however, is an effect of next order of approximation, and will not be considered here.

To estimate the possible increase of the cross section on nuclei up to 10^8 GeV it is necessary to calculate first the behavior of the pp cross section σ_{tot}^{pp} at such high energies. To main parameters of the model are the cross section $\overline{\sigma}$ and the average slope of the diffraction peak $\overline{a}$ for that part of the pp amplitude which gives the constant cross section, and also the analogous values σ_0 and a_0 for the meson-meson interaction. The expression for σ_{tot}^{pp} is[2]

$$\sigma_{tot}^{pp} \approx \overline{\sigma} + 4\pi \int_0^\infty b\,db \left(1 - \frac{\overline{\sigma}}{8\pi \overline{a}} e^{-\frac{b^2}{4\overline{a}}}\right)\left(1 - \exp\left[d_0 \ln^2\left(\frac{p_L}{p_0}\right) e^{-\frac{b^2}{4a_0}}\right]\right). \tag{1}$$

Two parameters in (1) ($p_0 \approx 65$ GeV and a combination of a_0 and d_0) can be fixed on the basis of data on the pp cross sections at ISR energies. There are, of course, no experimental data on the value of a_0. In order not to contradict the data on the slope of the diffraction peak at ISR data (although these data are rather indefinite), it is necessary to choose a slope parameter approximately half as large as $\overline{a}$ (in which case $d_0 \approx 0.03$). This value of a_0 agrees with the usually assumed slope of the $\pi\pi$ scattering. As to the energy dependences of $\overline{a}$ and a_0, we assume that these quantities remain approximately constant when the energy is increased beyond the limits of the ISR interval. At the assumed parameters, this corresponds to a rather weak energy dependence of the width of the pp-scattering diffraction peak. The assumption that $\overline{a}$ and a_0 increase logarithmically would lead to a more appreciable increase of the cross sections on nuclei at cosmic-ray energies. It is clear that the cross section σ_{tot}^{pp} calculated by us is only a rough estimate at ultrahigh energies. The purpose of the calculation that follows is to show that a model in which the phase shift increases in proportion to $\ln^2 p_L$ can lead in natural manner to an insignificant growth of the cross sections on nuclei at high energies.

The cross section σ_{tot}^{pp} calculated from formula (1) is shown in Fig. 1. In the calculation we used the values $\overline{\sigma} = 38.4$ mb and $\overline{a} = 2a_0 = 6$ (GeV/c)^2 at $p_L > 10^3$ GeV/c. The dashed line in the same figure shows for comparison the extrapolation of $\ln^2 p_L$ to higher energies. We see that the curve for σ_{tot}^{pp} lies much lower than the plot of $\overline{\sigma} + 0.5 \ln^2(p_L/65)$ mb.

The inelastic cross section for the nucleus was calculated in standard fashion in the Glauber approximation (additivity of the phase shifts for scattering by individual nucleons), which is similar in character to the eikonal approximation of the initial model. The nucleon density in the nucleus was parametrized by a Gaussian function, which yields sufficiently good results in the diffraction cone.[8,9] The calculation was carried out for ^{14}N nuclei. The rms nucleus of the radius, in accord with the low-energy data, was assumed equal to 2.48 F, the nucleon-nucleon interaction cross section was chosen in accordance with Fig. 1, and the energy dependence of the slope of the pp amplitude at $p_L > 10^3$ GeV/c was disregarded. To check on the calculation procedure, we calculated the inelastic cross section on ^{12}C nuclei at $p_L = 60$ GeV/c. It agreed within the limits of errors with the measured value 248 ± 2 mb.[10]

The calculation results are given in Fig. 2. The same figure shows the values for the effective inelastic scattering by air nuclei, $\sigma_{eff}^{pAir} \leqslant \sigma_{eff}^{pAir}$ at energies $10^2 - 3$

$\times 10^4$ GeV, in accordance with the reduction in[11] (the upper limits are not indicated) and with the data for the cross section at ultrahigh energies,[4–6] obtained from a study of extensive air showers (EAS). The latter data were taken in accord with the reduction of[7], where it was recognized that not every nucleon-nucleus collision generates an EAS, so that it is necessary to distinguish between $\sigma_{\text{inel}}^{p\text{Air}}$ and $(\sigma_{\text{inel}}^{p\text{Air}})_{\text{EAS}}$ calculated from the measurement of the proton range prior to the production of the EAS by it. The values of $\sigma_{\text{inel}}^{p\text{Air}}$ given in[7] are approximately 20% higher than without allowance for this effect. Taking the rough character of the calculations and of the cosmic-ray measurements into account, we can state that the curve of Fig. 2 is in satisfactory agreement with the measured $\sigma_{\text{inel}}^{p\text{Air}}$.

Thus, the nucleon interaction model in which the scattering phase shift increases in proportion to $\ln^2 p_L$ can be reconciled both with the rather rapid growth of the *pp* cross section at ISR energies and with the negligible growth of the proton-nucleus cross sections at ultrahigh energies, at least if it is assumed that the effective slope of the nucleon-scattering amplitude changes little at ultrahigh energies.

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