Thermoelectric phenomena in superconductors and thermomechanical circulation effect in a superfluid liquid

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The question of thermoelectric phenomena in superconductors is discussed. The possibility is indicated of observing the circulating thermomechanical effect in superfluid helium.

The unique features of thermoelectric phenomena in superconductors[1] are connected with the possible existence of a normal current of density $j_n = b\vec{\nabla}T$ and of a superfluid current j_s. At the same time, in an open circuit consisting of an isotropic and homogeneous type-I superconductor we have $\mathbf{j} = \mathbf{j}_s + \mathbf{j}_n = 0$, and the appearance of the thermal current $\mathbf{j}_n$ leads only to additional heat transfer characterized by a certain thermal-conductivity κ_c. We can see, however, that the role of κ_c is quite small.[2–4] In a homogeneous but anisotropic superconductor, when $\vec{\nabla}T$ and the symmetry axes do not coincide, complete cancellation of the currents $\mathbf{j}_n$ and $\mathbf{j}_s$ is no longer possible, and a certain resultant current $\mathbf{j}_s^{(0)}$ should appear and produce a magnetic field.[1,4] In an unevenly heated isotropic but inhomogeneous superconductor, an uncompensated current $\mathbf{j}_s^{(0)}$ should likewise appears, together with the corresponding field $\mathbf{H}_0$. The concrete case of a bimetallic plate, was discussed briefly in[1] and we shall dwell on it here in greater detail (Fig. 1a).

We start with the equations for the current densities

$$\mathbf{j}_n = b\vec{\nabla}T, \qquad \mathbf{j}_s = \frac{e\hbar N_s}{2m}\left(\vec{\nabla}\phi - \frac{2e}{\hbar c}\mathbf{A}\right). \tag{1}$$

Here $N_s/2$ is the concentration of the superconducting pairs, ϕ is the phase of the macroscopic wave function, $\psi = \sqrt{N_s/2}\,e^{i\phi}$, and $\mathbf{A}$ is the vector potential. By virtue of (1) we can also write

$$\operatorname{rot} \Lambda \mathbf{j}_s = -\frac{1}{c}\mathbf{H}, \qquad \Lambda = \frac{m}{e^2 N_s} = \frac{4\pi\delta^2}{c^2}, \tag{2}$$

where δ is the London depth of penetration of the field into the superconductor.

In the interior of the plate, where the metal is homogeneous (region outside the strip AB), we have $\mathbf{j} = \mathbf{j}_n + \mathbf{j}_s = 0$. The same holds true in the region of the junction, where $T = 0$. Therefore $\mathbf{j} = 0$ along the contour L indicated in Fig. 1a, and by virtue of (1) and (2) we have

$$\frac{4\pi}{c}\int_{T_1}^{T_2}[b_{\mathrm{I}}(T)\delta_{\mathrm{I}}^2(T) - b_{\mathrm{II}}\delta_{\mathrm{II}}^2(T)]\,dT = \oint \mathbf{A}_l\, d\mathbf{l} = \int H_n\, dS = \Phi, \tag{3}$$

where Φ is the magnetic-field flux through the contour L (since we are considering the singly-connected region $\oint_L \vec{\nabla}\phi\, dl = 0$). Obviously, the introduction of the phase ϕ does not play any role in the derivation of the result, and we can start directly from Eq. (2). This was indeed done in[1], where the obtained estimate for the field H_0 in a bimetallic plate agrees with that obtained from (3).

We consider now a bimetallic plate with an opening (Fig. 1b). Repeating the same reasoning for this case, we obtain the result (3) with Φ replaced by $\Phi - \Phi^{(0)}$ where $\Phi^{(0)} = \pi\hbar cn/e$, $n = 0, 1, 2, \ldots$, inasmuch as now $\oint_L \vec{\nabla}\phi\, dl = 2\pi n$. This result coincides with that obtained in[5,6] and obviously pertains not only to a bimetallic plate, but also to the quite general case of a closed circuit of two superconductors I and II. The corresponding flux was observed in[7]. For a homogeneous circuit, when $\delta_{\mathrm{I\text{-}I}}^2 = \delta_{\mathrm{II}}^2 b_{\mathrm{II}}$, the effect vanishes, of course. Yet it is indicated in[5] that the authors "disregard completely" the effects connected with the inhomogeneity of the superconductors. As is clear from the foregoing, one cannot agree with this statement, and the effect discussed in[1] is the same as in[5–7]. It is a different matter that a "circuit with an opening" was not considered in[1] and it is not our intent to belittle the interest and significance of the extension of the arguments in[5–7] to just

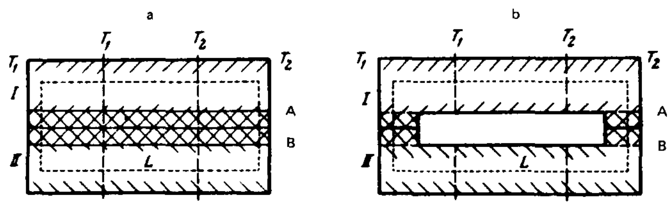

FIG. 1.

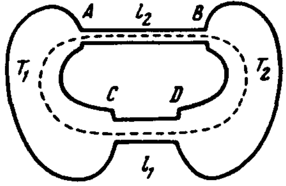

FIG. 2.

this geometry.[1)] In the case of a closed circuit there are also the questions of enhancement of the effect by connecting several circuits in series, of the situation in the case of an open circuit, and of the process of the evolution of the current $j_s^{(0)}$, the flow of which in the superconducting layer along the circuit leads to the appearance of the flux Φ. It is obvious that in the case of an open circuit there exist in the region of the discontinuity (e.g., in the gap) two superconducting regions in which the phase shifts ϕ_I and ϕ_{II} are different. Therefore when the circuit is closed there is produced, roughly speaking, a certain surface energy [proportional to $\int(\vec{\nabla}\phi)^2 dl$],[2)] and a superconducting current appears, the evolution of which can be analyzed only by using nonstationary equations (see[9,10] in this connection).

On the other hand, the well known analogy between superconductivity and superfluidity makes is possible to conclude immediately that a circulating thermomechanical effect can exist in a superfluid liquid. For the sake of argument, we consider an unevenly heated annular vessel filled with helium II (Fig. 2). Let in this case the sections AB and CD represent thin cylindrical channels of length l_1 and l_2 respectively and of cross section area S_1 and S_2. Then, by virtue of the presence of the potential difference $\Delta T = T_2 - T_1 = +\Delta P/\sigma\rho$, the normal and superfluid parts of the liquid circulate in the channels, and the velocity of the normal liquid averaged over the cross section is (see[11])

$$v_n^{(1,2)} = -\frac{\rho\sigma S_{1,2}}{8\pi\eta_n l_{1,2}}\Delta T. \qquad (4)$$

where ρ is the density of the helium, σ is the entropy per unit mass, and η_n is the corresponding viscosity coefficient. When the ring is opened, superfluid motion takes place only in the channels, with $v_s^{(1,2)} = -(\rho_n/\rho_s)v_n^{(1,2)}$. But if the ring is closed, circulation of the superfluid liquid should take place, with $\oint \mathbf{v}_s \cdot d\mathbf{l} = (\hbar/m_{He})2\pi n$. If $n=0$, in accord with the conditions for the circulation formation, then the velocity of the superfluid liquid in the uniformly heated part of the ring is

$$v_s(0) = -\frac{\Delta\phi_T}{L}\frac{\hbar}{m_{He}} = -\frac{\rho_n}{\rho_s}\frac{\rho\sigma(S_1 - S_2)\Delta T}{8\pi\eta_n L}, \qquad (5)$$

where L is the total length of the circuit (ring) and it is assumed that $l_1/S_1 + l_2/S_2 \ll L/S$, where S is the cross section of the main part of the ring. Indeed, the sections AB and DC, owing to the presence of flow with velocities $v_s^{(1,2)}$, lead to the appearance of a phase difference

$$\Delta\phi_T = \frac{m_{He}}{\hbar}(v_s^{(1)} l_1 - v_s^{(2)} l_2) = \frac{m_{He}}{\hbar}\frac{\rho_n}{\rho_s}\frac{\rho\sigma(S_1 - S_2)\Delta T}{8\pi\eta_n}$$

where ϕ is the phase of the wave function $\psi = \sqrt{\rho_s}e^{i\phi}$, so that $v_s = (\hbar/m_{He})\vec{\nabla}\phi$. To "cancel" the phase shift $\Delta\phi_T$ and to ensure the condition $\oint \mathbf{v}_s \cdot d\mathbf{l} = \oint(\mathbf{v}_s^{(1)} + \mathbf{v}_s^{(2)}) \cdot d\mathbf{l} = 0$, we must have a flow with velocity (5). Great interest attaches to the question of the evolution of the circulation of the flow at $n \neq 0$, etc. The circulation can be revealed in principle by the reaction of the walls of a freely suspended ring or a stack of rings. In our opinion, the effects discussed above are worthy of attention.

1)In a nonsuperconducting thermoelectric circuit (say for simplicity a wire bent to form a circle of radius r and consisting of two parts with different b, but with a constant conductivity σ throughout and with a cross section $S = \pi\rho^2$), the current is given by

$$I_n = \frac{\mathcal{E}_n}{R_n} = \frac{\rho^2}{r}\oint b\,dT,\quad \mathcal{E}_n = \oint\frac{b}{\sigma}dT,\quad R_n = \frac{2\pi r}{\sigma S} = \frac{2r}{\sigma\rho^2}.$$

When this circuit goes over into the superconducting state, however, according to (3), a current $I_s \sim (\delta^2/r)\oint b\,dT$ circulates in the circuit, inasmuch as $\Phi \sim \pi r^2 H$ and $H \sim (4\pi/c)(I_s/r)$. Thus, $I_s/I_n \sim (\delta/\rho)^2$ and in ordinary measurements (see, e.g., [8]) we have $I_s/I_n \ll 1$.

2)By way of an analogy we can indicate the junction of two magnetized ferromagnetic pieces in which the magnetization vector has different directions.

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