Singularities of vacuum decay and remarks on tachyons

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Theories in which the vacuum is not stable and a spontaneous local transition of vacuum into another state are presently under lively discussion.[1–4]

A strong argument against the instability of vacuum is aforded by cosmology, and specifically by the theory of the hot universe. Near a singularity, at extremal temperature and density, it is not very probable that a metastable state can remain intact. Under these conditions the following principle should hold: "everything that can happen has already happened."

Let us disregard cosmological considerations, however. The problem of spontaneous decay of vacuum is of general interest and has nontrivial singularities.

Instability of vacuum with respect to some process should be characterized by a quantity w, namely the probability that vacuum ceases to be vacuum and that some object is produced, say a bubble with a scalar quantity ϕ_1 against the background of a constant ϕ_2 in the entire surrounding volume, or else that a set of particles is produced. This quantity w should be referred to a unit (3-dimensional) volume and to a unit time, w cm^{-3} sec^{-1}. In a system of units with $\hbar = c = 1$ it can be stated that w has the same dimensionality as the energy density or the cosmological constant (divided by the gravitational constant)—see, e.g., [5].

In essence, w is the imaginary part of the cosmological constant. We can draw here a complete analogy with the quantum states of a system having a finite number of degrees of freedom: the stable ground state is characterized by energy; the excited state capable of decay is characterized by energy and by decay probability, which combine to form the concept of a complex energy.

The nontrivial difference between decay of vacuum and decay of a particle or of a material medium lies in the relativistic invariance of the vacuum. For this reason, as it turns out, w cannot be finite and a logically closed theory can be formulated only with account taken of external factors that violate Lorentz invariance (i.e., with allowance for the wall of the vessel or for the substance located in the vacuum). The produced system has $P_4 = E = \mathbf{p} = 0$ identical in all reference frames. This, however, does not exclude the possibility of objective determination of its velocity in a given reference frame. Nor does it exclude the definition of a reference frame in which the system is at rest. It does not include likewise differences in the magnitudes and directions of the velocities of the produced systems. Of course, the usual definition $\mathbf{v} = \mathbf{P}/E$ does not hold at $\mathbf{P} = E = 0$. However, for example, a produced bubble with $\phi = \phi_1$ represents a sphere of radius r, in which the energy density is negative (or at any rate lower than on the outside) and a thin expanding layer, the positive surface energy of which (with allowance for the relativistic velocity) compensates for the negative volume energy. The center of the bubble is at rest in only one reference frame, and its expansion is symmetrical. In this system $E_v = -E_s \neq 0$, $\mathbf{p}_v = \mathbf{p}_s = 0$ (V is the volume and S is the surface). In any other system we have $\mathbf{P}_v = -\mathbf{P}_s \neq 0$ and we can define $\mathbf{v} = \mathbf{p}_v/E_v = \mathbf{p}_s/E_s$ in spite of the fact that $E = E_s + E_v = \mathbf{p} = \mathbf{p}_s + \mathbf{p}_v \equiv 0$. If spontaneous creation of an object in vacuum is possible, then this object can just as rightfully have arbitrary velocity (less than unity, i. e., less than c) relative to a given coordinate frame. According to the formulas of special relativity theory, the integral in velocity space diverges—this integral is given by

$$I = 4\pi \int_0^1 \frac{v^2 dv}{(1 - v^2)^2} .$$

If in place of the velocity $v = d|r|/dt$ we introduce u $= dr/d\tau = v/\sqrt{1-v^2}$, so that $\mathbf{u}_4 = \gamma = 1/\sqrt{1-v^2}$, then we obtain

$$I = 4\pi \int_0^\infty \frac{u^2 du}{\sqrt{1+u^2}},$$

and the divergence remains (there is no limit such as $v < 1$ for $|\mathbf{u}|$). The fundamental theory should yield a definite differential probability of object production, referred to a volume element in velocity space near the origin

$$w_1 = dw/d^3u \Big|_{u<<1} = dw/d^3v \Big|_{v<<1} .$$

The total probability of the production of an object with arbitrary velocity is $w = w_1 I$, and consequently diverges. The assumption that w_1 tends to zero but in such a way that the product $w_1 I$ is finite does not eliminate the paradoxes: objects whose velocity is equal to unity are produced in any coordinate frame, since I diverges at $v = 1$. The momentum p and the energy E of the object as a whole are equal to unity in any frame, in particular at $v = 1$. But each object consists of at least two parts with $E_0 > 0$ and $E_0 < 0$ in the system $v = 0$. The energy of each part tends to infinity (in absolute value) at $v = 1$, a fact incompatible with observation. The divergence of I reflects the universally known noncompactness of the Lorentz group. We note in particular that the integration does not include, obviously, any u-dependent form factor. A detailed examination of bubble production yields exponentially small factors, owing to the presence of an energy barrier. The remark made above concerning the divergence of the integral means that ultrarelativistic bubbles should be produced; the proximity of their velocity to the velocity of light is limited by the same factors that violate the Lorentz invariance of the problem. The absolute value of the probability can remain small because of the smallness of the exponential in sub-barrier penetration[6]; the pre-exponential factor, in the system with $\hbar = c = 1$, is of the order of $m^4 \approx 10^{66}$ cm^{-3} sec^{-1} for m equal to the nucleon mass.

The significance of the arguments advanced above in tachyon theory[7] remains doubtful. The tachyon energy E_t is not positive-definite, and reverses sign under a Lorentz transformation. Therefore it is possible to construct out of two tachyons and two normal particles a system with $E = \mathbf{p} = 0$, capable of being produced in vacuum. For charged tachyons, the probability is unacceptably large, $\alpha^2 m^4 \sim 10^{62}$ cm^{-3} sec^{-1}even without integration over the velocities. However, the author of tachyon theory knew about this difficulty; he has noted that if the tachyon *goes out* of the production point 0 in a system where $E_t > 0$, then after going over to a system where $E_t < 0$ it must be stated that the tachyon enters into the point 0. Consequently, the process in question is a collision of two tachyons and not their spontaneous creation in vacuum. The original sin of causality violation by tachyons covers up other unusual conclusions of tachyon theory.

An argument against tachyons is that in the hot-universe theory it is possible to have statistical equilibrium for other particles, too.

If we nevertheless interpret the production of two tachyons and two ordinary particles as a spontaneous process, then the minimal probability is obtained for neutral tachyons, leaving only the gravitational interaction, $w_1 = G^2 m^6 \sim 10^{-10}$. Integration over the velocity, limited by collisions with other particles (photons, $n = 400$ cm^{-3} on the average in the universe) yields $w = G^2 m^{14} n^{-2} \sim 10^{70}$ cm^{-3} sec^{-1}. Consequently, this variant of the theory of neutral tachyons is unacceptable.

These considerations may turn out to be significant also in other variants of the scalar-field theory. On the other hand, the production of charge particles in a constant electric field, as is well known, has a finite probability, although it does lead to other paradoxes.[8]

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