# Light scattering by parametrically excited spin waves

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Raman scattering of light by magnetic-moment oscillations with parallel pumping in ferromagnets can be used to diagnose the state of a system of excited spin waves.

As shown in[1], in the case of parallel pumping of spin waves (SW) in ferromagnets, a stationary state of parametrically-excited magnons (PEM) can be realized, with the following relations satisfied[1]:

$$\langle a_{\mathbf{k}} \rangle = 0, \quad \langle a_{\mathbf{k}}^{+} a_{\mathbf{k}} \rangle = N_{\mathbf{k}}, \quad \langle a_{\mathbf{k}} a_{-\mathbf{k}} \rangle = \sigma_{\mathbf{k}} e^{-i\omega_p t}, \qquad (1)$$

where $\langle \cdots \rangle$ denotes averaging over the PEM ensemble, $a_{\mathbf{k}}^{+}$ is the operator of creation of a magnon with wave vector $\mathbf{k}$, and $\omega_p$ is the pump frequency; the density $N_{\mathbf{k}}$ of the PEM and $\sigma_{\mathbf{k}}$ depend on the pump amplitude and on the character of the magnon-magnon interaction. To explain the physical meaning of relations (1), we proceed to describe the PEM system in terms of waves of the magnetization $\mathbf{m}$ and compare it with two known SW distributions.

Inverting the Holstein-Primakoff (HP) and the Bogolyubov-Tyablikov (BT) transformation and changing over to a coordinate representation, we obtain from (1) the relations

$$\begin{aligned} &\langle m_{x,y}(\mathbf{r}t) \rangle = 0, \quad \langle m_z(\mathbf{r}t) \rangle = P_z + Q_z \cos(\omega_p t - \phi_z), \\ &\langle m_\alpha(\mathbf{r}t) m^*_\beta(\mathbf{r}t) \rangle = P_{\alpha\beta} + Q_{\alpha\beta} \cos(\omega_p t - \phi_{\alpha\beta}), \end{aligned} \qquad (2)$$

where $P_z$, $P_{\alpha\beta}$, $Q_z$, $Q_{\alpha\beta}$, $\phi_z$, and $\phi_{\alpha\beta}$ are real quantities, and are function of $N_{\mathbf{k}}$ and $\sigma_{\mathbf{k}}$ and of the HP and BT transformations; here and below, the $z$ axis of the employed system of coordinates is directed along the static magnetization vector $M$, and $\alpha, \beta = x, y$. We note that in the right-hand sides of (1) there is no dependence on the coordinate $\mathbf{r}$.

As is well known, the following relations hold for the thermal equilibrium of the SW noise:

$$< m_{x,\,y}(\mathbf{r}t)> = 0, \quad < m_z(\mathbf{r}t)>=\text{const}\,(\mathbf{r}t), \quad < m_\alpha(\mathbf{r}t)m^*_\beta(\mathbf{r}t)> = \text{const}\,(\mathbf{r}t). \tag{3}$$

It is possible to produce also a different SW distribution, with homogeneous or inhomogeneous magnetization oscillations, $\langle m_{x,y}(\mathbf{r}t)\rangle \neq 0$. Such a distribution is usually called coherent, and it can be represented in the form of a superposition of standing SW.

Comparing the characteristics of these three distributions, we see that the PEM system described by formulas (1) represents a noise distribution rather than a coherent distribution. Unlike thermal noise, however, in this case the mean values $\langle m_z\rangle$ and $\langle m_\alpha m_\beta^*\rangle$ depend on the time. (Such a noise distribution "breathes" so to speak at the pump frequency.)

Inelastic (Raman) scattering of light can be connected with magnetic[3] and electric[4] transitions. We take both processes into account.[2)] The scattering of light by PEM was calculated by us, following[3], with allowance for the dependence of the dielectric constant of the ferromagnet on the magnetization oscillations, $\epsilon_{ij} = \epsilon_0\delta_{ij} + if\epsilon_{ijk}m_k$ in the case of a transparent optically-isotropic medium ($\epsilon_{ijk}$ is an absolutely antisymmetrical tensor of third rank). Summation over repeated indices is implied throughout.

The differential cross section for the scattering of light by a unit volume of the ferromagnet is

$$\frac{d\epsilon}{d\Omega} = \frac{\omega^2}{2\pi c^4} S_{\alpha\beta}(\mathbf{n}) < m_\alpha(\mathbf{q}t)m^*_\beta(\mathbf{q}t) > , \tag{4}$$

where, if the incident light is unpolarized,

$$\begin{cases} S_{\alpha\beta} = S^e_{\alpha\beta} + S^m_{\alpha\beta} + S^{em}_{\alpha\beta}, \\ S^m_{\alpha\beta} = \frac{1}{2}(g\epsilon_0)^2\{\sin^2\theta\delta_{\alpha\beta} + \cos\theta(n_\alpha n^\circ_\beta + n_\beta n^\circ_\alpha)\} \\ S^e_{\alpha\beta} = \frac{1}{32\pi^2}(f\omega)^2\{n^\circ_\alpha n^\circ_\beta + n_\alpha n_\beta + (\mathbf{n}^\circ\mathbf{n})_\alpha(\mathbf{n}^\circ\mathbf{n})_\beta\} \\ S^{em}_{\alpha\beta} = \frac{1}{4\pi}(\epsilon_0\omega f g)(n_\alpha n^\circ_\beta + n_\beta n^\circ_\alpha), \qquad \cos\theta \equiv \mathbf{n}\mathbf{n}^\circ \end{cases} \tag{5}$$

and for the PEM described by relations (1) we can obtain

$$\begin{cases} < m_\alpha(\mathbf{q}t)m^*_\beta(\mathbf{q}t) > = a_{\alpha\beta}(\mathbf{q})N_\mathbf{q} + b_{\alpha\beta}(\mathbf{q})\sigma_\mathbf{q} e^{-i\omega_p t} + \text{c.c.} \\ a_{xx\,(yy)} = \frac{\mu_0 M_0}{2\hbar\omega_s(\mathbf{q})}(A_\mathbf{q} \mp \operatorname{Re}B_\mathbf{q}), \quad b_{xx\,(yy)} = \frac{\mu_0 M_0}{2\hbar\omega_s(\mathbf{q})}(\pm A_\mathbf{q} - B_\mathbf{q}) \\ a_{xy} = a_{yx} = 0 \qquad b_{xy} = b^*_{yx} = i\frac{\mu_0 M_0}{2}\,. \end{cases} \tag{6}$$

In formulas (4)—(6), $\mathbf{n}^0$ and $\mathbf{n}$ are unit vectors in the directions of the incident and scattered light, respectively, $\mathbf{q} = (\mathbf{n} - \mathbf{n}^0)/\lambda$, $\omega$ and $\lambda = c/\sqrt{\epsilon_0}\omega$ are respectively the frequency and wavelength of the incident light, $g$ is the gyromagnetic ratio, $d\Omega$ is the solid-angle element, $\mu_0$ is the Bohr magneton, $\omega_s(\mathbf{q})$ are the SW frequencies, and $A_\mathbf{q}$ and $B_\mathbf{q}$ are the coefficient of the nondiagonalized SW Hamiltonian, with $|B_\mathbf{q}| \ll A_\mathbf{q}$ (see[6]). We note that the quantity $S^{em}_{\alpha\beta}$ in formulas (5) is the result of interference between the "magnetically" and "electrically" scattered light.

It follows from (4) and (6) that the scattering cross section (i.e., the relative intensity of the light scattered into a given angle) varies periodically with time. This means that when relations (1) are satisfied, the light scattered by the PEM system is *modulated at the pump frequency*.

In order that the incoherent light flux be modulated in intensity at a certain frequency $\omega_1 - \omega_2$, it is not sufficient for it to consist of photons of frequency $\omega_1$ and $\omega_2$. In fact, let

$$\mathbf{E}_{1,2}(\mathbf{r}t) = \mathbf{e}_{1,2}\exp\left\{i\omega_{1,2}\left(\frac{x}{c} - t\right) + i\phi_{1,2}\right\} + \text{c.c.}, \tag{7}$$

where $\mathbf{e}_{1,2}$ and $\phi_{1,2}$ are random quantities, with $\mathbf{e}_{1,2}$ real positive amplitudes. Since the light is incoherent, the electric field intensity averaged over the ensemble of photons is $\langle \mathbf{E}_{1,2}(\mathbf{r}t)\rangle = 0$ and consequently $\langle \mathbf{e}_{1,2}\exp(i\phi_{1,2})\rangle = 0$. The intensity of this light flux is

$$I \sim < \mathbf{e}_1^2 > + < \mathbf{e}_2^2 > + < \mathbf{e}_1\mathbf{e}_2 e^{i(\phi_1 - \phi_2)} > \exp\left\{i(\omega_1 - \omega_2)\left(\frac{x}{c} - t\right)\right\} + \text{c.c.} \tag{8}$$

Consequently, the light is modulated when its mean value over the ensemble $\langle \mathbf{e}_1 \cdot \mathbf{e}_2 \exp[i(\phi_1 - \phi_2)]\rangle$ differs from zero or, introducing $\tilde{\mathbf{e}}_{1,2} \equiv \mathbf{e}_{1,2}\exp(i\phi_{1,2})$, when $\langle \tilde{\mathbf{e}}_1 \cdot \tilde{\mathbf{e}}_2^*\rangle \neq 0$.

The character of the distribution of the PEM in k-space determines the anisotropy of the scattering. If the PEM distributions are singular, like those obtained in[1], then the light scattering is strongly anisotropic. When the PEM are distributed over a sphere of radius $k_0$ k-space, so that

$$\sigma_\mathbf{k} = \frac{1}{4\pi k_0^2}\sigma_0\delta(|\mathbf{k}| - k_0)$$

and

$$N_\mathbf{k} = \frac{1}{4\pi k_0^2}N_0\delta(|\mathbf{k}| - k_0),$$

then the scattering by the PEM curves when the condition $k_0\lambda < 2$ is satisfied, and the scattered light is concentrated on the cone $\cos\theta_0 = 1 - k_0\lambda/2$. In the general case the light is scattered in those directions of $\mathbf{n}$ for which the values of $\mathbf{k} = \pm(\mathbf{n} - \mathbf{n}^0)/\lambda$ belong to the set of k-space points at which the singular distribution of PEM is concentrated.

When account is taken of two-magnon processes, the conservation laws of which do not impose such stringent limitations on the direction of the scattered light, there should be observed besides the scattering of light in individual directions also a uniform "background" of scattered light.

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1)Relations (1) are valid also for the PEM distribution obtained in[2].
2)According to Le Gall's calculations[5] the contributions of these two processes become comparable for YIG in the shortwave part of the optical spectrum.

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