Relaxation oscillations of parametric turbulence

V. V. Pustovalov, A. B. Romanov, V. P. Silin, and V. T. Tikhonchuk

P. N. Lebedev Physics Institute, USSR Academy of Sciences
(Submitted July 9, 1974)
ZhETF Pis. Red. **20**, No. 6, 356–359 (September 20, 1974)

It is shown that a magnetic field alters significantly the previously known laws of parametric-turbulence relaxation. Undamped periodic oscillations of the plasma-noise intensity can arise in the nearer-threshold region of parametric instability. Such a system can serve as a generator for relaxation oscillations of electric fields.

The nonstationary theory of parametric turbulence has revealed a number of unique regularities characterizing the relaxation of plasma fluctuations. Thus, nonlinear detuning of resonances, due to the dependence of the frequency of the electron Langmuir oscillations on their intensity, which plays a significant role in the case of small nonlinear parametric detuning, leads to a growth, monotonic in the time t, of the level of the plasma fluctuations excited by high-power radiation, [1] and to saturation of this level. To the contrary, the redistribution of the energy over the turbulence spectrum, due to the induced scattering of the waves, leads to establishment of a high level of turbulence with subsequent damped oscillations corresponding to a breakup of the turbulence scale[2,3] (cf. [4–6]). With an aim at revealing new qualitative regularities of nonstationary parametric turbulence, we report here the results of a theory of the evolution of plasma fluctuations produced in a magnetoactive plasma acted upon by a pump wave with an electric field intensity vector $\mathbf{E}_0$ parallel to the constant magnetic field $\mathbf{B}_0$, and with a frequency ω_0 close to the electron Langmuir frequency ω_{Le}. It turns out here that parametric decay of the pump wave is possible here into an oscillation but with the frequency of the upper hybrid resonance and into a slow magnetosonic wave, the conditions when the plasma frequency is large in comparison with the electron gyrofrequency Ω_e, and the frequency of the magnetic sound is small in comparison with the ion-rotation gyrofrequency Ω_i.

In the near-threshold region of parametric instability, when the ratio $p = E_0/E_{\text{thr}}$ of the pump-wave field intensity E_0 to the minimal threshold E_{thr} is larger than or of the order of unity ($p \gtrsim 1$), the spectral density $S(\theta, \tau)$ of the energy of the magnetic sound is determined by the solution of the nonlinear integro-differential equation

$$\frac{\partial S(\theta,\tau)}{\partial \tau} = \frac{S_0^2}{S(\theta,\tau)} + S(\theta,\tau)\left\{a^2 - \theta^2 + a\int_0^{\theta_0} \theta' d\theta' (\theta^2 - \theta'^2) \times \exp[-\beta^2(\theta^2 - \theta'^2)^2] S(\theta',\tau)\right\} \qquad (1)$$

in which 2θ is the angle at the apex of the cone of parametric buildup with axis along $\mathbf{B}_0$, $\tau = p^2(\pi/2)^{1/2}(\omega_{Li}/\omega_{Le})(\omega_0 - \omega_{Le})t$ is the dimensionless time, $\alpha^2 = 1 - p^{-2}$ is the excess over threshold, so is the dimensionless spontaneous noise, the value of the constant a yields a measure of the nonlinear interaction, and β determines the characteristic scale of the spectral redistribution[1)]:

$$S_0 = \frac{\pi}{6p}\frac{\nu_{ei}}{\omega_{Le}}\left[\frac{1}{9}\frac{{}^2{}_i}{\omega_{Le}^2} + \frac{2}{3}\frac{\omega_0 - \omega_{Le}}{\omega_{Le}}\right]^{-1/2};$$

$$\beta = \frac{1}{4\sqrt{2}}\frac{r_{De}}{r_{Di}}\frac{\Omega_e^2}{\omega_{Le}(\omega_0 - \omega_{Le})};$$

$$a = (32\pi^2 F^2 n_e r_{De}^3)^{-1}\frac{r_{Di}}{r_{De}}\frac{\Omega_e^2}{\omega_{Li}^3}(\omega_0 - \omega_{Le});$$

$$E_{\text{thr}}^2 = 2(2\pi)^{3/2} n_e \kappa T_e \frac{\nu_{ei}\,\omega_{Li}}{\omega_{Le}^2}.$$

At sufficiently large excess over threshold, $\alpha^2\beta \gg 1$, the solution of (1) conforms to the law of parametric relaxation with the turbulence level assuming a stationary value, in analogy with the results of[2,3]. The characteristic period τ_0 of the turbulent pulsations turns out

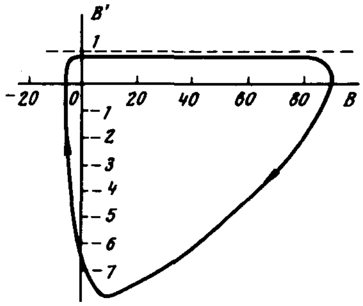

FIG. 1. Phase trajectory corresponding to Eq. (3), plotted for $\alpha = 10^{-2}$, $a^2 = 0.1$, and $\theta_0 = 1$. The arrows indicate the direction corresponding to increasing τ.

to be $\tau_0 = 2\pi\sqrt{3}(a^4\beta)^{-1}$, and the time τ_∞ of the total relaxation to the stationary state is given by the formula $\tau_\infty = 2a^8\beta^6(S_0^2 a^2\pi)^{-1}$.

A qualitatively new regularity appears in the relaxation of parametric turbulence in the case of a small excess over threshold $a^2\beta \ll 1$, when, assuming the fluctuation level to be on the average much higher than the spontaneous level, the solution of Eq. (1) can be approximately represented in the form

$$S(\theta,\tau) = S_0 \exp\left\{(a^2+\theta^2)B(\tau) - B'(\tau) + 1 - \frac{1}{2}aS_0\theta_0^2\right\}, \quad (2)$$

in which the function $B(\tau)$ satisfies the relation

$$(1-B')\exp(B') = \frac{a}{B}\exp(a^2B)[1-\exp(-B\theta_0^2)]; \quad B' \equiv \frac{dB(\tau)}{d\tau}. \quad (3)$$

Figure 1 shows the phase curve of B' against $B < \theta_0 \lesssim 1$. The fact that this curve is closed indicates that the solution of (3) is periodic and that by the same token the spectral distribution of the fluctuations (2) is periodically dependent on the time. Figure 2 illustrates the relaxation of the total energy density $S(\tau)$ of a slow magnetosonic wave with respect to time τ. At $0 \lesssim \tau \lesssim \tau_{max}$ the total noise increases relatively slowly from an initial spontaneous level S_0 to a maximum value $S_{max} = (2/a) \times \ln(\alpha a^2)^{-1}$. Further, at $\tau_0 > \tau > \tau_{max} \sim \tau_0 = a^{-2}\ln(a^2\alpha)^{-1}$ the total noise falls off rapidly (within the time $\tau \sim a^2$) to the initial level: the plasma returns from the turbulent state to the initial thermal state. Then ($\tau > \tau_0$) the described cycle is repeated with a period τ_0. The energy density averaged over the period, $\bar{S} = 2a^{-1}$, yields for the average electric field intensity $\bar{E}_S$ of a slow magnetosonic wave the expression

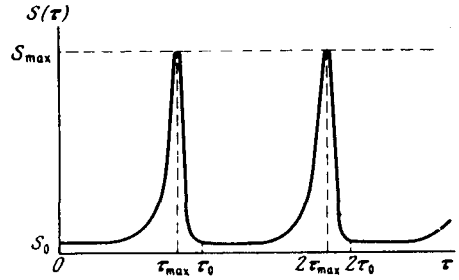

FIG. 2. Relaxation dependence of the total energy density $S(\tau)$ of a slow magnetosonic wave on the time τ at a small excess over threshold $a^2\beta \ll 1$ of the parametric instability; $\theta_0 = 1$.

$$\bar{E}_S = 8\sqrt{\pi}p\{n_e\kappa T_e\omega_{Li}(\omega_0 - \omega_{Le})\Omega_e^{-2}(r_{De}/r_{Di})\}^{1/2}.$$

The periodic relaxation oscillations of the plasma turbulence level, predicted in this communication, can be realized also for other types of parametric instability, when oscillations that grow with a corresponding increment can transfer their energy rapidly, owing to the nonlinear growth of the rate of spectral redistribution, into that region of wave-vector space in which they are damped.

The phenomenon described by us enables us to state that a parametrically unstable plasma can constitute a generator of relaxation oscillations of electric fields even in the near-threshold region.

[1)]The notation used by us is standard: n_e is the density, T_e is the plasma electron temperature (κ is the Boltzmann constant); ω_{Li} is the Langmuir frequency of the ions, $r_{De}(r_{Di})$ is the Debye radius of the electrons (ions), and ν_{ei} is the frequency of the Coulomb collisions of the electrons with the ions.

[1]V. V. Pustovalov, V. P. Silin, and V. T. Tikhonchuk, Zh. Eksp. Teor. Fiz. 66, 930 (1974) [Sov. Phys-JETP 39, No. 3 (1974)].

[2]N. E. Andreev, V. V. Pustovalov, V. P. Silin, and V. T. Tikhonchuk, ZhETF Pis. Red. 18, 624 (1973) [JETP Lett. 18, 366 (1973)].

[3]N. E. Andreev, V. V. Pustovalov, V. P. Silin, and V. T. Tikhonchuk, Kvantovaya élektronika, 1, 1099 (1974) [Sov. J. Quant. Electron. 4, 600 (1974)].

[4]V. E. Zakharov, S. L. Musher, and A. M. Rubenchik, ZhETF Pis. Red. 19, 249 (1974) [Sov. Phys.-JETP 19, 151 (1974)].

[5]Ya. I. Al'ber, Z. N. Krotova, N. A. Mityakov, V. O. Rapoport, and V. Yu. Trakhtengerts, Zh. Eksp. Teor. Fiz. 66, 574 (1974) [Sov. Phys.-JETP 39, No. 2 (1974)].

[6]W. L. Kruer and E. J. Valeo, Phys. Fluids 16, 675 (1973).