Estimate of the initial volume and multiplicity in the statistical theory of multiple production at high energies

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Starting from estimates of the effective longitudinal distances in deep-inelastic electroproduction processes and in hadron collisions, energy dependences of the initial volume and of the multiplicity, other than those usually assumed, are obtained in the hydrodynamic Landau theory.

In the Landau statistical-hydrodynamic theory of multiple production of particles at high energies,[1,2] an important parameter that determines the development of a process, is the initial volume of the system produced immediately after the collision. According to Landau, the hydrodynamic expansion after the collision proceeds adiabatically, and the multiplicity of the produced particles is proportional to the entropy S. If we assume[1,2] the ultrarelativistic equation of state of matter in this process to be $p=\epsilon/3$ (p is the pressure and ϵ is the energy density), then

$$S \sim VT^3, \tag{1}$$

where V is the volume of the system and T is its temperature at a given instant. By virtue of the entropy conservation in the expansion process and the energy conservation law $E_{cms}=(V_0)_{cms}\,T_0^4$, it follows from (1) that

$$n \sim S \sim E_{cms}^{3/4}\,V_{o\,cms}^{1/4}, \tag{2}$$

where V_0 is the volume of the system at the initial instant. It is usually assumed[1-4] that in hadron collisions (or, for example, in collisions between a nucleon and a real or virtual photon), at the initial instant the hadron can be regarded in its rest system as a sphere of radius $r\sim 1/m_0$, where m_0 is a certain mass of the order of 1–2 pion masses. Then, owing to the Lorentz contraction, the initial volume in the c. m. s., in the collision of two hadrons, is of the order of $V_0\sim(m/E_{cms})\,m_0^{-3}$, and according to (2) we have $n\sim E_{cms}^{1/2}\sim E_{lab}^{1/4}$. From the oblate shape of the initial volume in the c. m. s., it follows also that in the Landau hydrodynamic theory the produced particles have an angular distribution in the form of two jets.

The purpose of the present article is to call attention to the fact that the assumption indicated above concerning the shape and dimensions of the initial volume are not obligatory or even natural, and to point to another possibility and to the consequences that ensue from it.

We consider first the purest process from the theoretical point of view, that of deep-inelastic electroproduction on a nucleon, which can be treated as the process of the collision of a virtual photon with a mass squared $q^2<0$ and a nucleon (it is assumed that $|q^2|\gg m^2$, where m is the mass of the nucleon). Investigation of the space-time picture of the electroproduction process at large $|q^2|$ shows[5] that an important role is played in this process by transverse distances $\rho^2\sim 1/|q^2|$, and by the longitudinal distances in the laboratory frame $z\sim\nu/|q^2|\,m$ ($\nu=m(E-E')$, where E and E' are the initial and final energy of the electron). The growth of the longitudinal distances with energy of the virtual photon $q_0=\nu/m$ (at fixed q^2) follows[5,6] from the existing experimental data on electroproduction, and can be explained by assuming that the lifetime of the virtual photon in its rest system is $\tau\sim 1/\sqrt{|q^2|}$ (this estimate is obtained, e.g., in the parton model). Thus, in deep-inelastic electroproduction the initial volume in the laboratory frame is proportional to

$$V_{o\,lab.} \sim \nu/(q^2)^2 m, \tag{3}$$

and the initial volume in the c. m. s. at $\nu\gg m^2$ is

$$V_{o\,cms} \sim \frac{\nu}{(q^2)^2}\frac{E_{cms}}{\nu} = \frac{E_{cms}}{(q^2)^2}. \tag{4}$$

It follows from (2) and (4) that in deep-inelastic electroproduction the multiplicity n should be proportional to

$$n \sim \frac{E_{cms}}{\sqrt{|q^2|}} = \sqrt{\frac{q^2+2\nu+m^2}{|q^2|}} \tag{5}$$

in contradiction to the customarily assumed[4] $n\sim E_{cms}/\nu^{1/4}$. Relation (5) does not contradict the experimental data,[7] although a detailed comparison is made difficult by the fact that most existing data correspond to small q^2.

In the derivation of relations (4) and (5) we considered only the case of large $|q^2|\gg m^2$. The reason for this limitation is that at $|q^2|\gg m^2$ there exist peripherial interactions for which the Landau hydrodynamic theory does not hold. Therefore, when similar reasoning is applied to the case of pure hadronic pollutions, it is necessary, as incidentally always in such cases,[3] to have a criterion for separating the central collisions. If, however, we leave this fundamental question aside, then the arguments presented above can be directly applied to hadron collisions. The incident hadron dissociates virtually into different components, and according to the relation $\Delta E/\Delta t\sim 1$ the time of such a dissociation in the hadron rest system is $\Delta t\sim 1/M_{eff}$, where M_{eff} is the effective mass of the components. In analogy with the virtual photon, where this fact follows from experiment, we can expect M_{eff} not to increase with energy (although strictly speaking such a possibility is not excluded[8]). We then have in the laboratory system $\Delta t \sim (E_{1ab}/m)/M_{eff}$, the longitudinal distances are $z \sim \Delta t \sim (E_{1ab}/m)/M_{eff}$, and the initial volume is

$$V_{0\,1ab} \sim \frac{E_{1ab}}{m}\frac{1}{M^3_{eff}} \quad . \tag{6}$$

In the c. m. s., the initial volume, unlike the Landau case, is not oblate but prolate, and

$$V_{0\,cms} \sim \frac{E_{cms}}{mM^3_{eff}} \quad . \tag{7}$$

Thus, at the given estimate of the initial volume, the multiplicity is

$$n \sim E_{cms} \sim E^{1/2}_{1ab} \quad . \tag{8}$$

It is interesting to note that the dependence (8) of n on E coincides with the dependence obtained by Pomeranchuk[9] from entirely different considerations.

The picture of the hydrodynamic expansion produced with an initial volume (7) is not similar to the Landau hydrodynamic expansion. When it is compared with experiment it is necessary, as already mentioned, to have a selection criterion for central collisions. It is possible that such a criterion can be a selection of events in which at least one of the produced particles is characterized by large $p_\perp$.

We note in conclusion that in the case of e^+e^- annihilation into hadrons the considerations based on an estimate of the space-time distances that are important in the given process also lead[10] to an initial-volume estimate $V_0 \sim E^{-3}_{cms}$ and to a multiplicity $n \sim$ const, which are different from the customarily assumed $V_0 \sim m_0^{-3}$ and $n \sim E^{3/4}_{cms}$.

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