High-frequency spin-wave spectrum in ferromagnets

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(Submitted July 15, 1974)
ZhETF Pis. Red. **20**, No. 6, 375–378 (September 20, 1974)

The high-frequency spectrum of the spin waves due to the finite nature of the Larmor radius of the electrons is obtained within the framework of the electron-fluid theory.

The theory of the degenerate electron fluid of metals makes it possible to analyze consistently effects due to interaction of electrons and their motion. However, a relatively complete analysis of this type has been carried out only for normal metals (see, e.g.,[1]) and further development is still necessary for ferromagnets. In this communication we describe a theory of spin waves in a degenerate ferromagnetic electron fluid, connected with a consistent allowance for the orbital motion of the quasiparticles. The theory shows that high-frequency waves due to the finite Larmor radius of the electrons can exist in ferromagnetic metals.

Bearing in mind that the quasiparticle energy operator and the equilibrium density matrix depend on the spin

$$\hat{\epsilon}_0 = \epsilon_0(\mathbf{p}) - 2b(\mathbf{p})\hat{\mathbf{s}}\,\mathbf{m}, \quad \hat{n}_0 = \frac{1}{2}(n^+ + n^-) + (n^+ + n^-)\hat{\mathbf{s}}\,\mathbf{m}, \tag{1}$$

where $\hat{s}$ is the electron spin operator, m is a unit vector in the magnetization direction, $n^{\pm} = n_F(\epsilon_0 \mp b - \epsilon_F)$, $n_F(\epsilon)$ is the Fermi distribution function, and assuming the exchange energy b to be small in comparison with the Fermi energy ϵ_F, we can write down the following relation (cf.[2]) between the exchange energy and the constant B_0 of the Fermi-liquid interaction[3]:

$$B_0 = -\left[1 + \frac{1}{24}\left(\frac{b}{\epsilon_F}\right)^2 - \frac{\Omega_B}{\Omega_0}\right] \tag{2}$$

(it is assumed that the Fermi-liquid interaction can be approximated by one constant B_0, and that the quasiparticles have a quadratic dispersion law). Here $\Omega_B = 2\mu B/\hbar$ is the spin-resonance frequency of the electrons in the magnetic induction field B, and $\Omega_0 = 2b/\hbar$ is the exchange frequency.

In the magnetostatic approximation ($\omega \ll ck$), the spin-wave spectrum can be obtained with the aid of a kinetic equation for the density matrix. For waves polarized in a plane perpendicular to the magnetization direction, the kinetic equation leads to the following dispersion relation

$$\pm \frac{2\pi^2\hbar^3 v_F}{p_F^2} \sum_{n=-\infty}^{+\infty} \int \frac{d\mathbf{p}}{(2\pi\hbar)^3} J_n\left(\frac{k_\perp v_\perp}{\Omega}\right) \left\{ n_F\left[\epsilon_0(\mathbf{p} \mp \hbar\mathbf{k}_z) - \frac{v_F}{2p_F}(p_n^+)^2\right] - n_F\left[\epsilon_0(\mathbf{p}) - \frac{v_F}{2p_F}(p_n^-)^2\right]\right\} \left[\hbar\left(\omega \mp \Omega_0 \pm n\Omega + \frac{i}{\tau}\right) \pm \epsilon_0(\mathbf{p} \mp \hbar\mathbf{k}_z \mp \epsilon_0(\mathbf{p})\right]^{-1} = \frac{1}{B_0}. \tag{3}$$

The z axis is oriented here along the equilibrium magnetization, $J_n(x)$ is a Bessel function of the first kind, $k_\perp=\sqrt{k_x^2+k_y^2}$, $v_\perp=\sqrt{v_x^2+v_y^2}$, Ω is the Larmor frequency of the electrons, p_F and v_F are the Fermi momentum and velocity, and τ is the momentum relaxation time, which is assumed to be large throughout $(\omega\tau\gg 1)$. The upper and lower signs in the equations correspond to left- and right-polarized waves.

In the long-wave limit $(kv_F)^2\ll\Omega_0^2$, using condition (2), we can obtain from the dispersion relation (3) the well known low-frequency magnon spectrum[2]

$$\omega=\omega_M=\pm\left[\Omega_B+\frac{1}{12}\left(\frac{b}{\epsilon_F}\right)^2\frac{(kv_F)^2}{\Omega_0}\right]. \tag{4}$$

With increasing wave number, the low-frequency branch goes off to the region of Stoner excitations and is strongly damped. In contrast to the electron fluid of normal metal[1] and to the uncharged Fermi liquid,[3] where excitations of the zero-sound type are possible, there are no such excitations in ferromagnets.

Bearing in mind the singularities of the integral in (3), we conclude, in constrast to[4], that in addition to the low-frequency spectrum (4) there exists in each frequency interval $|\Omega_0-(n+1)\Omega\,|\omega|<|\Omega_0-n\Omega|$ solutions corresponding to a new type of excitations that propagate across the magnetization direction, or a special type of cyclotron waves.

The behavior of the frequencies of the spin cyclotron waves is similar in many respects to that of ordinary electron cyclotron waves. In the long-wave limit $(k_\perp v_F)^2\ll(n\Omega)^2$ the frequencies of the spin cyclotron waves are close to the resonant frequencies

$$\omega_n=\pm(\Omega_0-n\Omega) \tag{5}$$

of the transition of an electron between Landau levels that differ by the quantum number n, with spin flip

$$\omega=\omega_n-B_0R_n(k_\perp)\frac{n\Omega\epsilon_F}{\hbar(\omega_n-\omega_M)}, \tag{6}$$

where

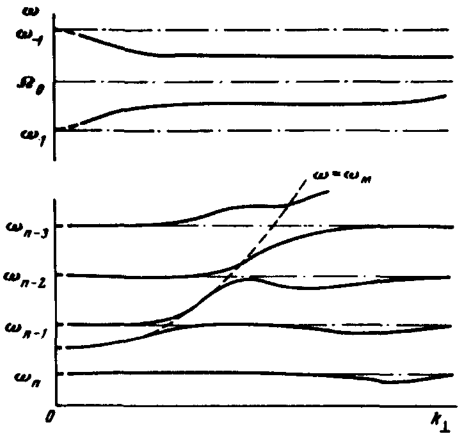
$$R_n(k_\perp)=a_n\left(\frac{k_\perp v_F}{\Omega}\right)\frac{4}{2n+3}\left[\left(\frac{p_n^+}{p_F}\right)^{2n+3}-\left(\frac{p_n^-}{p_F}\right)^{2n+3}\right],$$

$$a_n(x)=\frac{\sqrt{\pi}}{4}\frac{1}{n!\,\Gamma(n+3/2)}\left(\frac{x}{2}\right)^{2n}\sim\left(\frac{e}{2}\frac{x}{n}\right)^{2n}.$$

With increasing wave number, the differences $\omega-\omega_n$ increase monotonically. In the "short" wave limit $(k_\perp v_F)^2\gg(n\Omega)^2$ at not too large wave numbers, when

$$k_\perp v_F\ll\left(B_0^{-1}+\frac{p_0^++p_0^-}{2p_F}\right)^{-1}\omega_n,$$

the spin-wave frequencies are close to the arithmetic means of the resonant frequencies (5)



$$\omega=\pm\left[\Omega_0-\left(n+\frac{1}{2}\right)\Omega\right]-\frac{2\left(B_0^{-1}+\dfrac{p_0^++p_0^-}{2p_F}\right)}{\sum\limits_{j=0}^{\infty}(j+\frac{1}{2})^{-2}}\,\frac{k_\perp v_F\,\Omega}{|\Omega_0-(n+\frac{1}{2})\Omega|}. \tag{7}$$

With increasing wave number, the frequencies of the spin cyclotron waves again approach the resonant frequencies (5). If $k_\perp v_F\gg[B_0^{-1}+(p_0^++p_0^-)/2p_F]^{-1}\omega_n$, it is possible to obtain for them the asymptotic expression

$$\omega=\omega_n+\left(B_0^{-1}+\frac{p_0^++p_0^-}{2p_F}\right)^{-1}\frac{\Omega\omega_n}{2k_\perp v_F}. \tag{8}$$

Spin cyclotron waves, the spectrum of which is given by expression (6)—(8), are completely determined, in contrast to the electron cyclotron wave,[5,6] by inter-electron-correlation effects.

When the magnon frequencies ω_M approach the frequencies of the spin cyclotron waves (6), it is necessary to take into account the interaction of these modes. In this case we have

$$\omega=\frac{1}{2}(\omega_n+\omega_M)\pm\frac{1}{2}\sqrt{(\omega_n-\omega_M)^2-B_0R_n(k_\perp)\Omega_0\epsilon_F/\hbar}. \tag{9}$$

The spin-wave interaction given by (9) is due to correlation effects and is significant near the resonant frequencies $\pm(\Omega_0-n\Omega)$. It appears regardless of the electromagnetic interaction of the spin and ordinary cyclotron waves,[5] an interaction that takes place when the magnon frequency ω_M is close to the cyclotron harmonics $n\Omega$.

The behavior of the spin-wave frequencies is shown schematically in the figure.

We can thus state that the theory of a degenerate electron fluid reveals that spin cyclotron waves, due to the finite Larmor radius of the electrons, can exist in ferromagnets.

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