Contribution of paramagnons to the free energy of He^3

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The temperature dependence of the paramagnetic increment to the specific heat of normal He^3 was calculated and compared with experiment. The relative contribution of the paramagnons to the free energy of superfluid He^3 is estimated.

The specific heat of He^3 becomes linear only at temperatures lower than 0.1 °K, [1] which is much less than the Fermi energy. The strong deviation from the linear law can be explained by recognizing that He^3 is an almost ferromagnetic Fermi liquid. The temperature dependence of the specific heat can be expressed in terms of two adjustment parameters, one of which is obtained from the magnetic susceptibility and the other from its spatial dispersion. The same parameters determine the correction to the free energy of the superfluid state.

The exchange amplitude of the scattering of particles of an almost magnetic Fermi liquid is given by

$$\Gamma^k = -\frac{1}{1+F+(ak)^2}, \tag{1}$$

where the smallness of $1+F$ determines the proximity to the ferromagnetic transition, and k is the total momentum of the particle and the hole. The coefficient a is not expressed in terms of the Fermi-liquid parameters and is equal to the interatomic distance in order of magnitude.

In the calculation of the paramagnon increment to the free energy, it is necessary to sum ring diagrams that contain a maximum number of amplitudes Γ^k having one and the same k. As a result we obtain for the increment to the free energy

$$\delta F = \frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\,\mathrm{Sp}\,T\sum_{\omega}\ln\left(1+\Gamma^k\delta\hat{\Pi}(\omega, k)\right), \tag{2}$$

where $\delta\hat{\Pi}$ is the increment to the static value of the polarization operator and the trace is taken over the indices of the matrix $\delta\hat{\Pi}$.

In the normal state we have

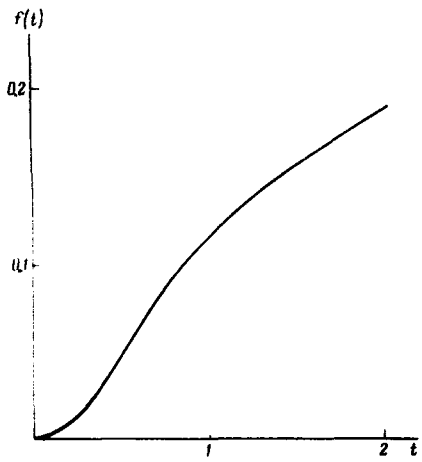
$$\delta\Pi_{ij} = -\delta_{ij}\pi|\omega|/2kv, \tag{3}$$

and formula (2) takes the form

$$\delta F = \frac{3}{2}\int\frac{d^3k}{(2\pi)^3}T\sum_{n}\ln\left[1+F+(ak)^2+\frac{\pi^2|n|T}{kv}\right]. \tag{4}$$

It is necessary to subtract from (4) the term that does not depend on the temperature and the term proportional to T^2, which gives the renormalization in the linear law of the specific heat. Expression (4) can be regarded as the free energy of a gas of Bose-type excitations—paramagnons.

After summing over n and differentiating with respect



to T we obtain for the total specific heat $C(T)$ the expression

$$\frac{C(T)}{T} = \frac{1}{3} m^* p_F \left[1 - \frac{1}{(a p_F)^2} f(t)\right]; \quad t = \frac{\pi^2 a T}{2(1+F)^{3/2} v} \tag{5}$$

The function

$$f(t) = \frac{9}{4} \int_0^\infty \left[\frac{1}{3} - \frac{\lambda t \operatorname{cth} \lambda t - 1}{\operatorname{sh}^2 \lambda t}\right] d\lambda \int_0^\infty e^{-\lambda(q+q^3)} q^2 dq, \tag{6}$$

which determines the relative contribution of the paramagnon to the specific heat, is shown in the figure.

At high temperatures we have

$$f(t) = \frac{1}{4}\left[\ln\frac{t}{\pi} + \frac{\zeta'(2)}{\zeta(2)} + \frac{3}{2} + \psi(1) + 2\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{2n-4}{3} \Gamma\left(\frac{n+1}{3}\right)\Gamma\left(\frac{2n+2}{3}\right)\zeta\left(\frac{2n-1}{3}\right)\left(\frac{1}{2t}\right)^{(2n+2)/3}\right], \tag{7}$$

where $\psi(x)$ is the logarithmic derivative of $\Gamma(x)$, and $\zeta(x)$ is the Riemann zeta function.

At low temperatures we have

$$f(t) = \frac{9}{4} t^2 \sum_{n=0}^{\infty} t^{2n} \frac{(3n+2)!\, 2^{2n+3} B_{n+2}}{n!\,(2n+2)!} \times \left[\ln\frac{\pi}{t} - \frac{\zeta'(2n+4)}{\zeta(2n+4)} - \frac{4n+7}{(2n+3)(2n+4)} - \frac{3}{2}\psi(3n+3) + \frac{1}{2}\psi(n+1)\right] \tag{8}$$

where B_n are Bernoulli numbers. The first term of this expansion makes a contribution of the form $T^3 \ln T$ to the specific heat, [2] but the low-temperature asymptotic form of the function $f(t)$ exists only in the region $t \lesssim 0.1$.

In the region from $t=0.2$ to $t=1$, the plot of $f(t)$ is, with good approximation, a straight line with slope 0.13. The experimental[1] value of $C(T)/T$, in the region from 0.01 to 0.05 °K, decreases linearly like $C(T)/T \sim (1-4.8\,T)$. Comparing the indicated slopes, we obtain

$$\frac{1}{a p_F (1+F)^{3/2} \epsilon_F} \approx 15\,°\mathrm{K}^{-1}. \tag{9}$$

Susceptibility experiments yield $1+F \approx 0.25$, so that

$$(a p_F)^2 \approx 0.36. \tag{10}$$

In the ladder approximation we have $(ap_F)^2 = 1/12$, but one cannot use the ladder approximation to calculate $(ap_F)^2$. More complicated diagrams as well as regions that are far from the Fermi surface make a large contribution, so that ap_F should be regarded as an adjustment parameter, and this is what we do.

At the indicated values of the parameters, the relation $C - \gamma T \sim T^3 \ln T$ holds true for temperatures less than one-hundredth of a degree. Owing to the superfluid transition, this region may not exist at all.

Formula (5) is valid so long as the relative value of the paramagnon increment to the specific heat is small, since no account of the dependence of the effective mass on the temperature and on the energy was taken in its derivation. In the experiment, the value of $C(T)/T$ changes by a factor of two when T changes from 0 to 0.1 °K, so that it is meaningless to compare formula (5) with experiment at $T > 0.05$ °K.

Let us discuss now the influence of paramagnon on the properties of superfluid He^3. The corresponding calculations were made by Brinkman *et al.* [3] The relative contribution of the paramagnons to the difference between the free energies takes in our notation the form

$$\delta = \frac{\Delta F^s_{BW}}{F_{AM} - F_{BW}} = \frac{150\,\pi^3}{56\,\zeta(3)} \frac{T_c}{\epsilon_F a p_F}\left(\frac{F}{1+F}\right)^{3/2}. \tag{10}$$

In[3] they used the value $(ap_F)^2 = 1/12$ for the estimates and obtained $\delta \approx 3$, which is much higher than the estimates of δ from the experimental data. Substituting our value $ap_F \approx 0.6$ we get $\delta \approx 1.5$, which agrees better with experiment.

[1]W. R. Abel, A. C. Anderson, W. C. Black, and J. C. Wheatley, Phys. Rev. **147**, 111 (1966).
[2]S. Doniach and S. Engelsberg, Phys. Rev. Lett. 17, 750 (1966).
[3]W. F. Brinkman, P. W. Anderson, and J. Serene, Preprint.