Rotation of the spin of particles in optically active media

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It is shown that in an optically active medium the particle polarization vector rotates in a manner that is kinematically analogous to the natural rotation of the plane of polarization of light.

Assume that a particle with spin moves through an optically active medium. The question: is the spin-polarization vector rotated about the direction of the particle motion, in kinematic analogy with the natural rotation of the polarization plane of light?

The possibility of rotation of the spin of the particle moving through an optically active medium is evident even from the classical model of an optically active medium,[1] in which the molecules are conducting helices. Assume that a charge particle moves along the axis of the helix with constant velocity. It induces in the helix a circular current, which in turn produces at the particle position a magnetic field parallel to the helix axis. It is clear that the particle spin will precess under the influence of this magnetic field.

Let us proceed to a more detailed examination of this phenomenon. In the general case of an optically-active isotropic medium, the dielectric tensor ϵ_{ij} is given by

$$\epsilon_{ij}(\omega, \mathbf{k}) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\epsilon^{tr}(\omega, \mathbf{k}) + \frac{k_i k_j}{k^2}\epsilon^l(\omega, \mathbf{k}) + if(\omega, \mathbf{k})e_{ijl}k_l,$$

(1)

where e_{ijl} is an antisymmetrical unit tensor and k_i are the components of the wave vector **k**. The magnetic field **B** generated at the position of the particle by the charge of a particle moving with velocity **v** can be obtained from Maxwell's equation and is given by

$$B_z = -\frac{e\beta^3}{2\pi^2}\int d^3k \frac{q^2 k_z^2 f(k_z v, \mathbf{k})}{(k^2 - k_z^2\beta^2\epsilon^{tr}(k_z v, \mathbf{k}))^2 - k_z^4\beta^4 k^2 f^2(k_z v,\mathbf{k})},$$

(2)

where $\beta = v/c$, $q^2 = k_x^2 + k_y^2$ and the z axis of the system is parallel to the velocity **v**.

We see that the magnetic field B_z at the particle

differs from zero only at $f \neq 0$, i.e., only in an optically-active medium.

For a concrete calculation of B_z, we use the fact that in the case of weak spatial dispersion[2] we have $f(k_z v, \mathbf{k}) \approx f(k_z v, 0)$ and $\epsilon^{tr}(k_z v, \mathbf{k}) \approx \epsilon^{tr}(k_z v, 0)$. Taking the foregoing into account, and introducing a new variable $k_z v = \omega$, we can integrate with respect to $d\omega$ by the method used in the theory of ionization losses,[3] and obtain in the approximation linear in f:

$$B_z = -\frac{ie}{c^3} \int_{\omega(0)}^{\omega(q_0)} \omega^2 \left(\frac{\epsilon^{tr}(\omega)}{c^2} - \frac{1}{v^2} \right) \frac{\frac{\partial}{\partial \omega}(\omega^2 f(\omega))}{\frac{\partial}{\partial \omega}\left(\omega^2 \left(\frac{\epsilon^{tr}(\omega)}{c^2} - \frac{1}{v^2} \right) \right)} \, d\omega. \quad (3)$$

The integration in (3) is with respect to the pure imaginary values of ω, and $\hbar q_0$ is the cutoff momentum. For the choice of the integration limits see[3].

We consider next two cases (cf.[3]):

1. We assume that the particle velocity satisfies the condition $v^2 > c^2/\epsilon_0$, where $\epsilon_0 = \epsilon^{tr}(0)$ is the electrostatic value of the dielectric constant. Using the microscopic theory of optical activity[1] we can write for $f(\omega)$ the expression

$$f(\omega) = \frac{1}{\omega^2} \sum_n \frac{g_{n0}}{\omega^2 - \omega_{n0}^2}, \quad (4)$$

where $g_{n0} = (16\pi c\rho/3\hbar)\omega_{n0}^2 R_{n0}$, ρ is the molecule density in cm^{-3}, $R_{n0} = \mathrm{Im}\langle 0|d|n\rangle\langle n|m|0\rangle$ is the rotational strength of the $n \to 0$ transition, d is the electric-moment operator, m is the operator of the magnetic dipole moment of the molecule, $\sum_n R_{n0} = 0$, and $\omega_{n0} = \omega_n - \omega_0$.

With the aid of (3) and (4) we can obtain

$$B_z = \frac{2e\sqrt{1-\beta^2}}{3c^3 \sqrt{\frac{4\pi Ne^2}{m}}} \sum_n g_{n0}. \quad (5)$$

2. We assume now that the condition $v^2 < c^2/\epsilon_0$ is satisfied. In this case an explicit expression for B_z can be obtained if

$$|\epsilon^{tr}(\omega) - 1| \ll \frac{c^2}{v^2}\left(1 - \frac{v^2}{c^2}\right).$$

As a result we have

$$B_z = -\frac{evq_0}{2c^3\sqrt{1-\beta^2}} \sum_n \frac{g_{n0}}{\frac{v^2 q_0^2}{1-\beta^2} + \omega_{n0}^2} + \frac{e}{2c^3} \sum_n \frac{g_{n0}}{\omega_{n0}} \arctan \frac{vq_0}{\omega_{n0}\sqrt{1-\beta^2}}. \quad (6)$$

As $v \to 0$, B_z tends to zero like v^3. At v close to c we have

$$B_z \approx \frac{\pi e}{4c^3} \sum_n \frac{g_{n0}}{\omega_{n0}}. \quad (7)$$

In this case the field B_z is constant and does not depend on the particle energy.

The fact that the magnetic field at the particle differs from zero leads to precession of the particle spin at the frequency (in the particle rest system)

$$\Omega = 2\mu B_z / \hbar.$$

The angle θ through which the spin rotates over the length l is therefore, in the laboratory frame,

$$\theta = \frac{2\mu B_z}{\hbar} \sqrt{1-\beta^2} \frac{l}{v}. \quad (8)$$

Thus, the angle through which the spin rotates over a length l first increases as the velocity increases from zero, and then again tends to zero at high energies as $v \to c$.

Let us estimate the magnitude of the effect. Using for the rotational strength R_{n0} of the transition the value $R_{n0} \approx 0.6 \times 10^{-37}$,[1] $\sqrt{1-\beta^2} \sim 10^{-1}$, $\mu = \mu_{\mathrm{electron}}$ and $\omega_{n0} \approx 10^{17}$ we obtain for the angle of rotation per unit length the estimate $\theta = 10^{-5}$ rad/cm.

We note in conclusion that in principle the particle spin precession occurs not only when the particle moves in an optically-active medium, but also in any medium that rotates the photon polarization plane (e.g., as a result of the Faraday effect, paramagnetic rotation, rotation of the γ-quantum polarization plane in a polarized electron target[4] and in a target with polarized nuclei, particularly Mössbauer nuclei).[5]

[1]W. Kauzmann, Quantum Chemistry, Academic, 1957.
[2]V. P. Silin and A. A. Rukhadze, Élektromagnitnye svoĭstva plazmy i plazmopodobnykh sred (Electromagnetic properties of plasma and plasmalike media), Atomizdat (1961).
[3]L. D. Landau and E. M. Lifshitz, Élektrodinamika sploshnykh sred (Electrodynamics of Continuous Media), Fizmatgiz (1959) [Pergamon, 1959].
[4]V. G. Baryshevskii and V. L. Lyuboshitz, Yad. Fiz. 2, 666 (1965) [Sov. J. Nucl. Phys. 2, 477 (1965)].
[5]V. G. Baryshevskii, Yad. Fiz. 4, 1045 (1966); 6, 714 (1967) [Sov. J. Nucl. Phys. 4, 749 (1967); 6, 520 (1968)].

Erratum: Rotation of the spin particles in optically active media
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In formula (3) on p. 191, the ratio

$$\frac{\frac{\partial}{\partial \omega}\left(\omega^2 f(\omega)\right)}{\frac{\partial}{\partial \omega}\left(\omega^2\left(\frac{\epsilon^{\prime r}(\omega)}{c^2}-\frac{1}{v^2}\right)\right)}$$

should be replaced by

$$\frac{\partial}{\partial \omega}\left[\frac{\omega^2 f(\omega)}{\frac{\partial}{\partial \omega}\left(\omega^2\left(\frac{\epsilon^{\prime r}(\omega)}{c^2}-\frac{1}{v^2}\right)\right)}\right].$$

Eq. (5) should be multiplied by 3/2, in Eq. (6) it is necessary to multiply the second term by 2 and replace v by v^3, and Eq. (7) should be multiplied by 2.