Transverse scaling at low energies

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It is shown that the dependence on the transverse mass $m_\perp = \sqrt{m^2 + p^2}$ in two-reggeon vertices $f_{a_1 a_2}(m_\perp)$, which appear in the description of the central region of inclusive spectra, can be accounted for within the framework of the multiperipheral model with an exponential internal cutoff. At $m_\perp > 0.4$ GeV, the relation between the $f_{a_1 a_2}$ is such that the complete answer for the cross section has a transverse-scaling form.

The Mueller-Regge approach[1] to the investigation of inclusive processes, supplemented with the hypothesis that the leading singularities can be factored and duality holds in the mean, explains many characteristics of inclusive spectra in the fragmentation region.[2,3]

It was shown in recent papers[4–9] that the Mueller-Regge representation for the cross section in the central region of the inclusive spectrum ($x \approx 0$) can explain the experimental data in a wide total-energy and rapidity range for a large class of inclusive reactions. In the limit of high energies, the differential cross section of the process $a + b \to c + X$ can be expressed in the indicated region in the form

$$E \frac{d^3\sigma}{d^3p} = \delta_P^a \delta_P^b \left\{ f_{PP}(m_\perp) + \frac{f_{RP}(m_\perp)}{\sqrt[4]{S}} (\gamma_a e^{y/2} + \gamma_b e^{-y/2}) + \frac{f_{RR}(m_\perp)}{\sqrt{S}} \gamma_a \gamma_b \right\} \quad (1)$$

Here $f_{pp}(m_\perp)$, $f_{RP}(m_\perp)$, and $f_{RR}(m_\perp)$ are unknown vertex functions, the values of which were determined earlier[5] by fitting the experimental data to formula (1) in one of the reactions, $\delta_i^{a,(b)}$ are the constants for the coupling between the reggeon i and the particle a, (b)

$$\gamma_{a,(b)} = \frac{\sum_{i \neq P} \delta_i^{a,(b)} r_i}{\delta_P^{a,(b)}},$$

where $\delta_i^{a,(b)}$ are so normalized that

$$\sigma^{ab}_{tot} = \delta^a_P \delta^b_P + \Sigma r_i\, \delta^a_i \delta^b_i \left(\frac{S}{S_o}\right)^{-1/2}.$$

As shown earlier,[5] $f_{\alpha\beta}(m_\perp)$ is approximated by simple exponentials in $m_\perp$, with close parameters of the logarithmic slope of the cone $B_{\alpha\beta}$, and it turns out that $B_{pp} > B_{RP} > B_{RR}$. A similar difference in the dependence on $m_\perp$ has made it possible to explain[6] the broadening of the cone in $m_\perp$ with increasing energy in the reactions $pp \to \pi^\pm X$, $\pi^\pm p \to \pi^\pm X$, and $K^+p \to \pi^- X$, the growth of the average transverse momentum with increasing $x = 2p_L/\sqrt{S}$, and the stronger violation of scaling with increasing $m_\perp$.

In this article we attempt to illustrate a similar difference in the dependence on $m_\perp$ at the vertex functions, on the basis of the multiperipheral picture of the interaction. If we use the expression derived by Barnett and Silverman[10] for the inclusive cross section in the central region, then we have in the high-energy limit the representation

$$\frac{d^2\sigma}{dp_\perp^2\, dy} = \sum_{\alpha_1\alpha_2} \frac{\lambda_{\alpha_1\alpha_2}}{S} \int d\omega_1 d\omega_2 B_1(\omega_1)\, B_2(\omega_2)\, \pi \frac{\Gamma(\alpha_1+1)\Gamma(\alpha_2+1)}{8(\omega_1+\omega_2)^{\alpha_1+\alpha_2+2}}$$

$$\times (\omega_1 S_1)^{\alpha_1} (\omega_2 S_2)^{\alpha_2} \exp\left\{\frac{2\omega_1\omega_2}{\omega_1+\omega_2} m_c^2\right\} e^{-k} k^{-\alpha_1}\, \Psi(\alpha_2+1, -\alpha_1+\alpha_2+1; k), \tag{2}$$

where

$$m_\perp^2 = p_\perp^2 + m_c^2 = S_1 S_2/S; \quad k = \frac{2\omega_1\omega_2}{\omega_1+\omega_2} m_\perp^2 = b\, m_\perp^2,$$

$$B_i(\omega_i) = \int_0^\infty \beta_i^2(t) e^{\omega_i t} dt; \quad S_1 \approx \sqrt{S} m_\perp e^y; \quad S_2 \approx \sqrt{S} m_\perp e^{-y}$$

$\beta_i(t)$ is a propagator corresponding to internal cutoff with respect to t in the multiperipheral model. In the case of a simple exponential cutoff we have $B_i(\omega_i) = \delta(\omega_i - \omega_i')$ and (2) takes the form

$$\frac{d^2\sigma}{dm_\perp^2\, dy} = \sum_{\alpha_1\alpha_2} \frac{\lambda'_{\alpha_1\alpha_2}}{S} S_1^{\alpha_1} S_2^{\alpha_2} e^{-k} k^{-\alpha_1}\, \Psi(\alpha_2+1, -\alpha_1+\alpha_2+1; k). \tag{3}$$

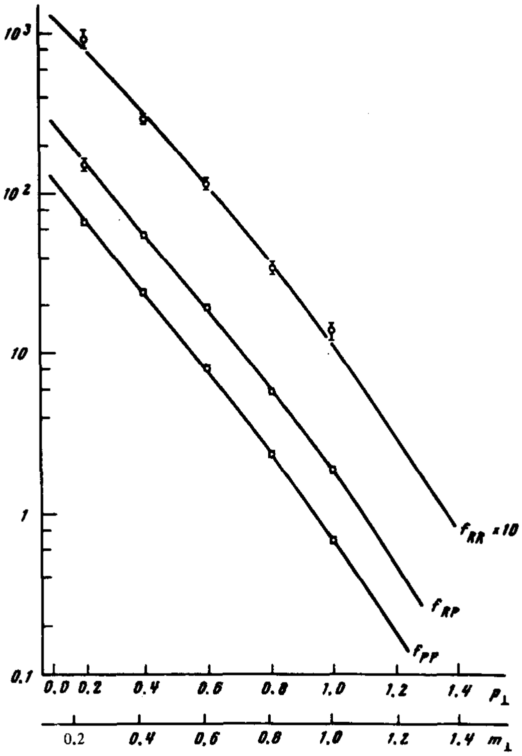
(All the constants connected with ω'_1 and ω'_2 are incorporated in $\lambda'\alpha_1\alpha_2$.) Comparing expressions (3) and (1) we easily see that the $f_{\alpha_1\alpha_2}$ from (1) can be expressed in the form

$$f^{MP}_{\alpha_1\alpha_2}(m_\perp) = \text{const}\; k^{\frac{\alpha_2-\alpha_1}{2}} e^{-k}\, \Psi(\alpha_2+1, \alpha_2-\alpha_1+1; k). \tag{4}$$

The only parameter that determines the dependence on m (besides the general normalization of each term in (3)) in all $f_{\alpha_1\alpha_2}$ is the coefficient $b = 2\omega_1\omega_2/(\omega_1+\omega_2)$.

In the case $\alpha_1 = 1$ and $\alpha_2 = 1$, we obtain the well known expression for the vertex function of the PP term

$$f_{PP}(m_\perp) = -(1+k)Ei(-k) - e^{-k}.$$



The figure shows the experimentally obtained values of $f_{\alpha_1\alpha_2}(m_\perp)$[5] in comparison with those calculated on the basis of formula (4) with the parameter value $b = 2.05$. We see that the $f_{\alpha_1\alpha_2}$ determined from the multiperipheral model at the indicated value of the parameter b describe well the dependence on $m_\perp$ in all the terms of the Mueller-Regge expansion.

In the region of large $m_\perp$, the asymptotic form of $f_{\alpha_1\alpha_2}(m_\perp)$ from (4) is

$$f_{\alpha_1\alpha_2}(m_\perp) \approx \text{const}\; e^{-k} k^{-(\alpha_1+\alpha_2+2)/2} \approx \text{const}\, \frac{e^{-b m_\perp^2}}{m_\perp^{\alpha_1+\alpha_2+2}}.$$

The asymptotic value coincides then with the exact expression from (4) at $m_\perp > 0.4$ GeV, and expression (3) takes the form

$$\frac{d^2\sigma}{dm_\perp^2\, dy} = \text{const}\, \frac{e^{-b m_\perp^2}}{m_\perp^4} \{1 + \beta_1\sqrt{ze^y} + \beta_2\sqrt{ze^{-y}} + \beta_1\beta_2 z\}, \tag{5}$$

with β_1 and β_2 connected with γ_i by the relation

$$\beta_1 = C\gamma_a; \quad \beta_2 = C\gamma_b.$$

It follows from (5) that, at least at $m_\perp > 0.4$, the energy-dependent part of the inclusive-process cross section can be written in a scaling form (relative to the transverse momentum). Thus, the scaling law of the type $\phi(m_\perp) f(z)$, observed at $p_\perp > 2$ GeV/c and $y = 0$ in the ISR experiment,[11] holds true in the central region down to small $m_\perp$ and low energies. The differential cross section for $y \neq 0$ can be written in factorized form

$$\frac{d^2\sigma}{dm_\perp^2\,dy} = \phi(m_\perp)\Psi_1(u)\,\Psi_2(v),$$

where

$$u = \sqrt{z}\,e^{y/2};\quad v = \sqrt{z}\,e^{-y/2};\qquad z = \frac{m_\perp}{\sqrt{S}}$$

and at large y we have $u \approx \sqrt{x}$.

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