Feasibility of generation of high-power hypersound with the aid of laser radiation

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A nonlinear equation was obtained for the description of the generation of high-power hypersound by mixing light waves and SMBS at low temperatures, when it is necessary to take the acoustic nonlinearity into account. The saturation regime is investigated; the limiting intensities of the hypersound are determined. A dimensionless number A and a nonlinear length z_{nl}, which determine the dynamics of the process, are introduced.

1. We propose in this paper a correct method of calculating an intensive hypersound field.

Up to now it was customary, when describing laser generation of hypersound, to use a linear equation for the density increment p[1–3]; the reason is the large value of the absorption coefficient α. At temperatures lower than 20 °K, α decreases to the very small value $\sim 10^{-3}$ cm^{-1} and it is necessary to take into account the terms nonlinear in p, a fact pointed out many times.[2–4]

2. By using the method of slowly varying profiles[5] it is possible to obtain from the complete system of hydrodynamic equations (elasticity theory) an equation never before encountered in wave theory:

$$\frac{\partial p}{\partial z} - \frac{\epsilon}{c_0^3 \rho_0} p \frac{\partial p}{\partial \tau} - \frac{b}{2c_0^3 \rho_0} \frac{\partial^2 p}{\partial \tau^2} = \frac{Y}{16\pi} E_1 E_2 \frac{\Omega}{c_0} \sin \Omega\tau. \qquad (1)$$

Here z is the longitudinal coordinate, $\tau = t - z/c_0$, c_0 is the speed of sound, ρ_0 is the density of the medium, b is the effective viscosity, Y is the parameter of the optical-acoustic coupling, and ϵ is the parameter of the acoustic nonlinearity. We have confined ourselves for simplicity to the case when two opposing laser beams of close frequencies ω_1 and ω_2 and amplitudes E_1 and E_2 interact, and the optical dispersion of the medium permits synchronous excitation of sound at the difference frequency $\Omega = \omega_1 - \omega_2 = 2n\omega_{1,2}c_0/c$.

To analyze (1), it is convenient to change over to the dimensionless variables

$$\theta = \Omega\tau, \qquad x = \alpha z = \frac{b\,\Omega^2}{2c_0^3 \rho_0} z, \qquad \Pi = \frac{2\epsilon p}{b\,\Omega}. \qquad (2)$$

The meaning of the coordinate x is the distance in damping lengths, while Π is the running Reynolds number. Equation (1) takes the form

$$\frac{\partial \Pi}{\partial x} - \Pi \frac{\partial \Pi}{\partial \theta} - \frac{\partial^2 \Pi}{\partial \theta^2} = A \sin\theta. \qquad (3)$$

The number

$$A = \frac{\epsilon Y}{16\pi} \frac{E_1 E_2}{c_0^2 \rho_0} \left(\frac{\Omega}{c_0 \alpha}\right)^2 = \frac{2\epsilon Y \sqrt{I_1 I_2}\, n}{c_0^2 \rho_0 c \alpha^2} \left(\frac{2\pi}{\lambda}\right)^2 \qquad (4)$$

(where $I_{1,2}$ are the intensities and λ is the wavelength of the light) serves as a convenient criterion for the appearance of nonlinear acoustic effects. Low-amplitude hypersound is excited at $A \ll 1$ ($\Pi \ll 1$), and the nonlinearity can become appreciable at $A \gg 1$.

3. At small A we seek the solution of (3) by the method of successive approximations, so that we obtain expressions for the amplitudes 1 and 2 and for the higher harmonics:

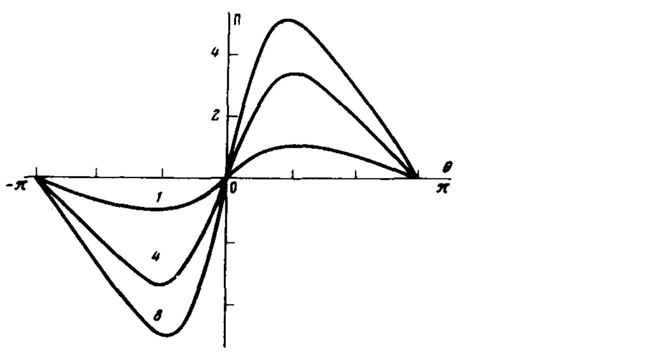
$$\Pi^{(1)} = A(1 - e^{-x})\sin\theta, \qquad \Pi^{(2)} = \frac{A^2}{8}(1 - e^{-x})^3 \left(1 + \frac{1}{3}e^{-x}\right)\sin 2\theta. \qquad (5)$$

We note that at small x the second-harmonic amplitude increases in accordance with the cubic law $\sim A^2 x^3/6$. As $x \to \infty$, the amplitudes tend to constant values, thus indicating the presence of a stationary regime in the system. The solutions (5) allow us to estimate the limits of the linear approximation. At $(A/8) \ll 1$ it describes satisfactorily the process for arbitrary x. When this condition is violated, the nonlinear effects can be disregarded up to $x_{nl} = (6/A)^{1/2}$ or

$$z_{nl} = \frac{\lambda}{2\pi}\left(3c_0^2 \rho_0 c/n\epsilon Y \sqrt{I_1 I_2}\right)^{1/2}. \qquad (6)$$

The "nonlinear length" z_{nl} serves as a characteristic scale of the onset of saturation.

4. In the region where the nonlinear effects are strong, it is necessary to seek an exact solution of (3). It can be obtained, but the solution and the analysis are cumbersome and are not presented here. We write out only the expression for the stationary wave:



Stationary profile of wave at different values of the number A.

$$\Pi = 2\frac{\partial}{\partial\theta}\ln ce_0\left(\frac{\theta}{2}, \frac{A}{8}\right), \qquad -\pi \lesssim \theta \lesssim \pi . \qquad (7)$$

Profiles having the same period $\Pi(\theta)$ are shown in the figure for the values $A = 1$, 4, and 8. It is easy to see that the steady-state wave form is not of the sawtooth type; it is not quite correct to apply the results of free sound propagation to the case of excitation with light, for the exchange between harmonics proceeds in a different manner in this case. The maximum hypersound intensity is determined by the eigenvalue $\gamma(A)$ corresponding to the Mathieu function $ce_0(\theta/2, A/8)$, and equal to

$$I_{ac} = \frac{4|\gamma| c_0^5 \rho_0 \alpha^2}{\epsilon^2 \Omega^2} . \qquad (8)$$

In the case of low A we have $4|\gamma| \approx A^2/2$ and Eqs. (8) and (4) lead to the obvious relation $I_{ac} = (Y^2\omega^2/2c^4 c_0 \rho_0 \alpha^2) I_1 I_2$. At large A we obtain from (8) a more complicated dependence of I_{ac} on the intensity of the light waves ($A \to \infty$, $I_{ac} \sim \sqrt{I_1 I_2}$).

5. Let us discuss the conditions for experimentally observing the nonlinear regime (7). It is obviously necessary to stipulate $z_{nl} \ll \alpha^{-1}$ and $A \gg 1$, and furthermore $z_{nl} \lesssim c_0\tau_L$, where τ_L is the duration of the exciting laser pulses. The length z_{nl} (6) depends little on the temperature and is determined mainly by the intensity of the laser emission. By focusing a giant pulse one can obtain $I \sim 10^6$ MW/cm^2 and decrease z_{nl} to $\sim 10^{-4}$ cm. Thus, the requirement $z_{nl} \ll \alpha^{-1}$ can be satisfied even at high temperatures. On the other hand, the pulse duration of a Q-switched laser suffices also to satisfy the stationarity condition $z_{nl} \lesssim c_0\tau_L$. Thus, even at the present status of the physical experiments one can observe intense hypersound that is saturated by an acoustic nonlinearity.

6. In Eq. (1) we can regard E_1 and E_2 as not specified by the fields of the incident radiation and the Stokes wave. Then, supplementing (1) with the equation $\partial E_{1,2}/\partial z = \sigma_{1,2}\tilde{p}E_{2,1}$, where $\tilde{p}$ is the amplitude of the first harmonic of p, we obtain a system for the description of SMBS in the case of backward scattering.

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