Spontaneous vacuum transitions in dual models

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It is shown that spontaneous vacuum transitions exist in a dual model with trajectory intercept $\alpha_0 = -1$.

As shown in[1,2], summation over the induced vacuum transitions in dual amplitudes leads to the following redistribution of the n-point B functions:

$$B_n^R(\beta, p_1, \ldots, p_n) = \sum_{N_i=0}^{\infty} \beta^{N_1+\ldots+N_n} B_{n+N_1+\ldots+N_n}(p_1, \overbrace{0,\ldots,0}^{N_1}, p_2, \overbrace{0,\ldots,0}^{N_2}, p_3, \ldots, p_n, \overbrace{0,\ldots,0}^{N_n}), \tag{1}$$

where β is the constant of the induced transition of the particles into the vacuum, and $B_m(p_1, \ldots, p_m)$ are dual amplitude in the Koba-Nielsen representation[3]

$$B_m(p_1, \ldots, p_m) = \frac{1}{\Omega} \int \frac{\prod_i dx_i}{\prod_i (x_{i+2} - x_i)} \prod_{\substack{i,j=1 \\ i<j}}^{m} (U_{i,j})^{-\alpha_{i,j}-1} \tag{2}$$

In formula (2), x_i denotes an ordered set of integration variables from $-\infty$ to $+\infty$; Ω is the invariant volume for the three variables and

$$U_{i,j} = \frac{(x_j - x_i)(x_{j+1} - x_{i-1})}{(x_j - x_{i-1})(x_{j+1} - x_i)}$$

are invariant anharmonic ratios.

The presence or absence of spontaneous vacuum transitions of particles in a vacuum is connected with the analytic properties of $B_n^R(\beta, p_1, \ldots, p_n)$ as functions of the parameter β. In the case when $B_n^R(\beta, p_1, \ldots, p_n)$ are multiply-valued functions of the parameter β, the value $\beta = 0$ on different sheets of these function corresponds to different sets of dual amplitudes, connected with the initial ones by spontaneous vacuum transitions.

To ascertain the analytic properties of the functions $B_n^R(\beta, p_1, \ldots, p_n)$ we have considered a dual model with a trajectory intercept $\alpha_0 = -1$. In this case the summation in the formula can be carried out exactly and leads to a redetermination of the trajectories of the initial model and to the appearance of new trajectories.

The principal Regge trajectory shifts by an amount 2α in comparison with the trajectory of the initial model. For external particles located on the shifted principal trajectory, the scattering amplitudes take the form

$$B_n^R(p_1, \ldots, p_n) = \left(\frac{\beta}{a}\right)^n \frac{1}{\Omega} \int \frac{\prod_i dx_i}{\prod_i (x_{i+2} - x_i)} (U_{i,j})^{-\alpha_{i,j}-2a-1}, \tag{3}$$

$$\alpha = \frac{1}{\pi} \arcsin \pi\beta. \qquad (4)$$

Starting with the second daughter trajectory, additional trajectories appear, shifted by an amount -2α relative to the initial one. The scattering amplitude for external particles on the thus-shifted trajectory are obtained from (3) by making the substitution $q \to -\alpha - 1$. Particles on trajectory shifted by 2α and by -2α cannot go over into one another.

The factor $(\beta/\alpha)^n$ in (3) corresponds to a renormalization of the wave functions of each of the external particles. If the particle wave functions are normalized to unity, the corresponding factor should also be replaced by unity.

The spontaneous vacuum transitions that appear in the model under consideration are connected with multiple-valuedness of the function (4). A zero value of the parameter β for the function (4) corresponds to an infinite set of values of the quantity α:

$$\alpha = n \qquad n = 0, \pm 1, \pm 2, \ldots,$$

each of which corresponds to definite sets of dual amplitudes.

At $n = 0$ and $n = -1$ the principal trajectory in the scattering amplitudes has a minimum intercept, $\alpha_0 = -1$. At $n = 1$ and $n = -2$, the intercept of the principal trajectory becomes equal to unity. On going from the case with $n = 0$ to the case with $n = 1$, the square of the particle mass reverses sign, in conformity with the analogous situation in the $\lambda\phi^3$ theory.

With further increase of n, a corresponding increase takes place in the intercept of the principal trajectory.

It appears that an analogous situation obtains also in other dual models.[1)]

[1)]A preliminary analysis of a dual model with an arbitrary intercept α_0 leads to the following relation for the values of α and β:

$$\frac{\Gamma(-\alpha_0)}{\Gamma(-\alpha)\Gamma(\alpha - \alpha_0)} = \beta$$

(sf. formula (4)).

[1]D. V. Volkov, A. A. Zheltukhin, and A. I. Pashnev, Yad. Fiz. 18, 902 (1973) [Sov. J. Nucl. Phys. 18, 467 (1973)].
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[3]Z. Kohn and H. B. Nielsen, Nucl. Phys. **B10**, 637 (1970); E. Dohini and S. Sciuto Ann. Phys. (N.Y.) 58, 388 (1970).