

Condensation in neutron stars

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We investigate the change introduced by π condensation in the equation of state of neutron matter.

The question of the phase transition in which a π condensate is produced in nuclear matter was considered in⁽¹⁾. It was shown later^(2,3) that in a neutron medium at a density $n = n_c^0$ the π^0 -meson field becomes unstable and a π^0 -condensate is produced. At approximately the same density, $n = n_c^+ \sim n_c^0$, there appears one more instability, namely there appears in the spectrum of the $\pi^+\pi^-$ -mesons a point where the sum of their energies vanishes. This instability leads to the formation of an electrically neutral condensate of $\pi^+\pi^-$ meson pairs. The appearance of condensates lower the energy of the

ground state of the system and restores its stability, i. e., the frequencies of the π^0 -mesons and the sum of the energies of the π^+ and π^- -mesons are positive after the appearance of the condensates.

It follows from our calculations (see⁽³⁾) that the critical densities n_c^0 and n_c^+ are close, but their ratio depends entirely on the values of the nucleon spin-spin interaction constants g^{nn} and g^- , which are unknown for a neutron medium ($n_c^0 = 0.4 \approx n_c^+$ at g^{nn} and g^- equal to the corresponding constants for a medium with $N=Z$, $g^{nn} = 1$, and $g^- = 0.8$).

We consider in this paper a neutron medium with density $n > n_c^0, n_c^*$. Assuming for simplicity that $n_c^0 = n_c^* = n_c$ and confining ourselves to perturbation theory in terms of the condensate field, we find the change produced in the equation of state by the π -condensate and the structure of the condensate field.

In the presence of π -condensate, the energy density of a system of neutrons at densities close to n_c can be expressed in the form ($\hbar = m_\pi = c = 1$)^[1]

$$E(n) = E_0(n) - \frac{\gamma}{2} (n - n_c)^2 \quad n \geq n_c, \quad (1)$$

where n is the neutron density and $E_0(n)$ is the neutron energy in the absence of the condensate. The second term represents the decrease in the neutron-system energy due to the π -condensation. Thus, the change in the equation of state near the transition point is determined by the constant γ .

The structure of the π -condensate and the value of γ depend on the values of the constants g^m and g^* of the nucleon spin-spin interaction, and also on the interaction of the π^0 -condensate and of the condensate of the $\pi^+\pi^-$ pairs. We shall first estimate the value of γ , assuming the condensates to be noninteracting. In this case $\gamma = \gamma_{\pi^0} + \gamma_{\pi^+\pi^-}$, where γ_{π^0} describes the contribution of the π^0 -condensate to the energy of the system and $\gamma_{\pi^+\pi^-}$ describes the contribution of the condensate of the $\pi^+\pi^-$ pairs.

A convenient method of ascertaining the structure of the static field $\phi(\mathbf{r})$ of the π^0 condensate and of estimating the value of γ_{π^0} is the Thomas-Fermi approximation in which $\phi(\mathbf{r})$ is assumed to be long-wave, i.e., $k_0^2/4p_F^2 \ll 1$ (k_0 is the wave number of the π^0 -condensate field).

As we show in^[5], in this case we have

$$\gamma_{\pi^0} = 0.6(1 + 0.9g^m)^4 \left(1 + \frac{1}{3} \frac{\left\langle \left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 + \left(\frac{\partial \phi}{\partial z} \right)^2 \right\rangle}{\left\langle \left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 + \left(\frac{\partial \phi}{\partial z} \right)^2 \right\rangle} \right)^{-1}. \quad (2)$$

The angle brackets denote here averaging over the coordinates, and the factor $(1 + 0.9g^m)^4$ is the result of allowance for the nucleon correlations.

As seen from this expression, the largest γ_{π^0} corresponds to a condensate field $\phi(\mathbf{r})$ having the form of a three-dimensional lattice

$$\phi(\mathbf{r}) = a(\cos k_0 x + \cos k_0 y + \cos k_0 z), \quad a^2 \sim n - n_c. \quad (3)$$

In this case

$$\gamma_{\pi^0}^{(3)} = 0.4(1 + 0.9g^m)^4. \quad (4)$$

By way of example, we present the values of γ_{π^0} for the field ϕ in the form of plane layers and a two-dimensional structure:

$$\phi = a \cos k_0 x, \quad \gamma_{\pi^0}^{(1)} = \frac{25}{27} \gamma_{\pi^0}^{(3)}, \quad (4')$$

$$\phi = a(\cos k_0 x + \cos k_0 y), \quad \gamma_{\pi^0}^{(2)} = \frac{50}{51} \gamma_{\pi^0}^{(3)}.$$

The contribution of the $\pi^+\pi^-$ -pair condensate to the system energy (i.e., to the quantity $\gamma_{\pi^+\pi^-}$) can be estimated in the following manner: Assume that the field of the condensate is given by

$$\phi(\mathbf{r}, t) = |a_1 \cos(k_1 r - \omega_1 t); a_1 \sin(k_1 r - \omega_1 t); 0| \quad (5)$$

(we recall that the meson field $\vec{\phi} = \{\phi_1, \phi_2, \phi_3\}$ is connected with the fields of the π^+ , π^- , and π^0 mesons in the following manner: $\phi_{\pi^+} = (\phi_1 + i\phi_2)/\sqrt{2}$; $\phi_{\pi^0} = \phi_3$). Then, as shown in^[4], the effective Lagrangian of the pion field in the neutron medium can be expressed (in the approximation in which $|\omega_1| \gg k_1 v_F$) in the form

$$L_\pi = L(\phi) - L(0) = (\omega^2 - \omega_k^2) \frac{a_1^2}{2} - \frac{n\omega}{2} (\xi - 1), \quad (6)$$

where $\omega_k = 1 + k^2$; $\xi^2 = 1 + 4f^2 k^2 a_1^2 / \omega^2$; $f = 1$ or 0 is the πN -interaction constant. Variation of L_π with respect to a_1 leads to an equation for a_1 ^[4]:

$$\omega^2 = \omega_k^2 + \frac{2nf^2 k^2}{\xi \omega}. \quad (7)$$

The electric-neutrality condition, as explained in detail by us in^[3], leads to the following equation for the determination of ω_1 ^[4]:

$$\frac{\partial L_\pi}{\partial \omega} = 0 \quad \text{or} \quad 2\omega \left[1 + \frac{\omega^2 - \omega_k^2}{\omega^2 (\xi + 1)} \right] a_1^2 = 0. \quad (8)$$

The wave number k_1 is determined from the condition that there be no current in the ground states of the system (see^[5])

$$\frac{\partial L_\pi}{\partial k} = 0 \quad \text{or} \quad \left(1 + \frac{2nf^2}{\xi \omega} \right) a_1^2 = 0. \quad (9)$$

Using the equality (see^[3])

$$E_\pi = \omega \frac{\partial L_\pi}{\partial \omega} - L_\pi$$

and taking (6)–(8) into account, we easily obtain expressions for the energy E_π and for the condensate-field amplitude a_1

$$E_\pi = -\frac{1}{2}(n - n_c)^2 \quad \text{i.e., } \gamma_{\pi^+\pi^-} = 1, \quad (10)$$

$$a_1^2 = \frac{1}{4}(n - n_c).$$

It turns out here that $\omega_1 = -1$ and does not depend on the density; $k_1^2 = (n/n_c) + 1$ and $n_c^* = (1/2f^2)n_c$.

Expression (10) was obtained under the assumption that $|\omega_1| \gg k_1 v_F$ and that the field of the $\pi^+\pi^-$ -condensate is of the form of a traveling wave (without assuming smallness of the condensate field). In^[5], without assuming that $|\omega_1| \gg k v_F$, we considered other field configurations. From among these, the field (5) gives the lowest energy. We show in the same paper how the result (10) is altered by taking nucleon correlations into account. We present here the value of γ_{π^\pm} , which turns out in this case to be

$$\gamma_{\pi^\pm} = 2.1 (1 + 0.25 g^-)^4 \quad (11)$$

So far we have disregarded the interaction between the π^0 -meson condensate and the condensate of the $\pi^+\pi^-$ pairs, which influences significantly the structure of the condensate field and the value of γ . A detailed discussion of this question is contained in^[5]. The main results obtained in this case are the following:

1. If the nucleon spin-spin interactions constants g^{nn} and g^- satisfy the inequality

$$g^{nn} < 2.3 + 0.8 g^- \quad (12)$$

then a π^0 -meson condensate and a $\pi^+\pi^-$ -meson pair condensate exist in the system simultaneously. The most convenient in this case is a field in the form $(\mathbf{k}_0 \| \mathbf{k}_1)$

$$\vec{\phi}(r) = \{ a_1 \cos(\mathbf{k}_1 \mathbf{r} - \omega_1 t); \quad a_1 \sin(\mathbf{k}_1 \mathbf{r} - \omega_1 t); \quad a_0 \cos \mathbf{k}_0 \mathbf{r} \},$$

for which we have

$$\gamma = 1.1(\gamma_{\pi^0}^{(1)} + \gamma_{\pi^\pm}) - 0.6 \sqrt{\gamma_{\pi^0}^{(1)} \gamma_{\pi^\pm}}. \quad (13)$$

2. If $g^{nn} > 2.3 + 0.8 g^-$, then only a static π^0 -condensate exists in the neutron medium. The condensate field takes the form of the three-dimensional lattice (3), and the corresponding quantity $\gamma = \gamma_{\pi^0}^{(3)}$ is given by (4). This condensate stabilizes the system, and no condensation takes place.

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