Physical cause of back bending in the plot of the moment of inertia against the rotation frequency

Yu. T. Grin'

I. V. Kurchatov Atomic Energy Institute
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It is shown that the cause of the back bending in the plot of the moment of inertia against the rotation frequency is a restructuring of the ground-state wave function due to the appearance of two-quasiparticle excitations. Expressions are obtained for the rotation frequencies at which the back bending takes place in the ground and β-vibrational bands.

Much attention is being paid at present to the back bending (b.b.) in the plot of the moment of inertia against the rotation frequency, first observed experimentally by Johnson *et al.* [1] There is no unambiguous explanation of the cause of this phenomenon. The most widely assumed are the following: 1) abrupt vanishing of pair correlation at a certain frequency[1]; 2) alignment of the angular momenta of a pair of free nucleons under the influence of rotation. [2] The first cause is connected with the possibility of a first-order phase transition in the rotating nucleus at an angular momentum I_{cr}, when the nucleus goes over completely from the superconducting to the normal state. In this case the pair correlation Δ vanishes jumpwise, [3,4] and the moment of inertia increases abruptly to its rigid-body value. The transition is the result of the fact that at $I \geqslant I_{cr}$ the normal state is energywise favored. In this approach, all the nucleon pairs in the nucleus are identical and are in no way distinguished from one another.

This situation, however, does not conform to the real structure of the nucleus, in which the nucleons occupy states with different angular momenta, and consequently behave differently with respect to rotation of the nucleus, since the Coriolis interaction of the nucleons depends on their angular momentum. There is therefore no first-order phase transition under the influence of the rotation of the nucleus. Actually, the particles of the nucleus interact with the rotation differently, depending on the single-particle angular momentum $\mathbf{j}$. The Coriolis interaction is large for nucleons with large angular momenta, since it is proportional to $\mathbf{I}\cdot\mathbf{j}$, and causes the gap Δ of precisely these particles to vanish first. A realignment takes place in the occupation of the states by the particles, free particles (not connected by pair correlation) appear at the levels near the Fermi surface, and their additional contribution to the moment of inertia can cause the latter to increase abruptly. We can propose a simple physical explanation for this phenomenon, based on energy considerations. The energy consumed in breaking the particle pair equals $E_1 + E_2$ (where $E_\lambda = \sqrt{\Delta^2 + (\epsilon_\lambda - \epsilon_0)^2}$, ϵ_λ is the energy of the single-particle level, and ϵ_0 is the energy of the Fermi surface). On the other hand, owing to the interaction between the angular momentum of the nucleon with the total angular momentum of the entire nucleus, we can obtain an energy gain $2\omega\sqrt{\langle M_x^2\rangle}$, where ω is the angular velocity of the nuclear rotation and $\langle M_x^2\rangle$ is the mean-squared projection of the angular momentum on the rotation axis. Thus, at a certain rotation frequency $\omega_1 = (E_1 + E_2)/2\sqrt{\langle M_x^2\rangle}$ there are produced in the nucleus excitations whose formation does not call for energy expenditure. With further increase of the rotation frequency ω, the occupation of the particles with this two-quasiparticle excitation becomes energywise favored. This phenomenon corresponds to the pair-breaking effect under the influence of rotation, first pointed out by the author and by Larkin in[5]. The remaining particles continue to remain in the superconducting bound state, and the total vanishing of Δ takes place not jumpwise, but extends over a certain angular-momentum region, leading to a second-order phase transition. [5] This circumstance refines and confirms the second point of view concerning the origin of the b.b., explaining the appearance of free particles on the Fermi surface. We note that the complete vanishing of Δ takes place at angular momenta $I \approx (20-24)\hbar$, [5] which greatly exceeds those of the b.b. We note that the appearance of such excitations becomes manifest in experiment as a crossing of rotational levels of the ground (or vibrational) and two-quasiparticle bands.

Let us estimate the angular velocities ω_1 at which gapless excitations appear first, as well as their dependence on the nuclear-level parameters. For a qualitative analysis we confine ourselves to the Gor'kov equations, first for the case when the off-diagonal part $\tilde{\Delta}=0$, and second in the approximation of three equidistant levels, i.e., we take into account exactly the off-diagonal parts of G and F between the levels (1, 2) and (2, 3). These approximations do not change the qualitative nature of the phenomenon and enable us to obtain an analytic expression with which to estimate the point where the gapless two-quasiparticle excitation appears, and to ascertain its dependence on the properties of the level on the Fermi surface. In this case ($\epsilon_1-\epsilon_2=\epsilon_2-\epsilon_3=d$, $\lambda=2$) we have

$$\omega_1(l_\lambda m_\lambda)=\sqrt{\frac{\Delta^2+d^2}{(M_x^2)_\lambda}}=\sqrt{\frac{2(d^2+\Delta^2)}{l_\lambda(l_\lambda+1)-m_\lambda^2}}. \qquad (1)$$

For a spherical nucleus, when $d\neq 0$ and $(M^2)_\lambda\approx l_\lambda^2$, this result was first obtained in[5].

It follows from (1) that a gapless two-quasiparticle excitation is produced first of all in nuclei for which a level with large l_λ and small m_λ lies on the Fermi surface, and for which d is also small, i.e., at the start of the filling of the shells $i_{13/2}$ and $i_{11/2}$ in the rare earths.

At the typical values $\beta\approx 0.3$, $\Delta\sim d\sim 0.9$ MeV, and $l=6\hbar$, both formulas give close values, $\omega_{1\,\mathrm{theor}}\approx 0.28$ MeV. This is very close to the experimentally observed angular velocity of the start of the b.b., $\omega_{1\,\mathrm{exp}}=0.29$ MeV. In nuclei with $N=90$, 92, 94, and 96 nucleons there are present on the Fermi surface (or close to it) levels from the $i_{13/2}$ shell with projections $1/2^+$, $3/2^+$, and $5/2^+$. This fact, in conjunction with the small $d=\epsilon_{3/2}-\epsilon_{1/2}$, determines the b.b. at the observed frequencies. When the number of nucleons is large, both the number m_λ and d increase, and this increases ω_1 by 1.5—2 times. No such frequencies have been observed in experiment as yet.

In the region of heavy elements, levels of the shell $j_{15/2}$ lie near the Fermi surface, but with relatively large projections, so that the distance d is large. An estimate based on formula (1) yields for this region $\omega_{1(15/2),m_\lambda=7/2}\approx 0.24$ MeV. Frequencies of the same order should be preferably observed at $I\sim 16\hbar$. Formula (1) and the physical considerations on which it is based can be easily extended also to b.b. of vibrational states, particularly β-vibrational states. In this case the minimal excitation energy is $E_1+E_2-\omega_\beta$, and formula (1) takes the form

$$\omega_{1\beta}\approx\frac{E_1+E_2-\omega_\beta}{2\sqrt{\langle M_x^2\rangle_\lambda}}=\frac{\sqrt{d^2+\Delta^2-\frac{\omega_\beta}{2}\sqrt{2}}}{\sqrt{l_\lambda(l_\lambda+1)-m_\lambda^2}}. \qquad (2)$$

Applying the estimate (2) to the nuclei Gd^{184} and Dy^{156}, where $\omega_\beta=0.680$ MeV and 0.674 MeV, respectively, we obtain $\omega_{1\,\beta\,\mathrm{theor}}=0.22$ MeV, which agrees with the experimental value $\omega_{1\,\beta\,\mathrm{exp}}=0.22$ MeV.[6]

Thus, the b.b. phenomenon in the ground state is connected with the realignment of the vacuum of the quasiparticles and with the appearance of a large admixture of certain two-quasiparticle states in the wave function of the ground state (lowest in energy). In the case of the β-vibrational state, on the other hand, the b.b. is connected with a transition from the vibrational state to a two-quasiparticle state. This circumstance should manifest itself in experiment as an abrupt change in the constant of the interaction between the ground-band levels and the levels of the β-vibrational band before and after the point $\omega_{1\beta}$.

[1] A. Johnson, H. Ryde, and S. Hjoth, Nucl. Phys. **A179**, 753 (1972).
[2] F. Stephens and R. Simon, Nucl. Phys. **A183**, 257 (1972).
[3] B. Mottelson and J. Valatin, Phys. Rev. Lett. **5**, 511 (1960).
[4] Yu. T. Trin', Zh. Eksp. Teor. Fiz. **14**, 445 (1961) [Sov. Phys.-JETP **41**, 320 (1962)].
[5] Yu. T. Grin' and A. Larkin, Yad. Fiz. **2**, 40 (1965) [Sov. J. Nucl. Phys. **2**, 27 (1966)].
[6] H. R. Andres, D. Ward, R. Graham, and J. Geiger, Nucl. Phys. **A219**, 141 (1974).