The vertex constants of the α particle

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The vertex constants for the virtual decays $\alpha \to T+N$ and $\alpha \to d+d$ are calculated by solving the integral equations for four nucleons. The vertex constants in a model with separable spin-dependent Yamaguchi potential are $G^2_{\alpha TN} = 17.5 \pm 0.9$ F and $G^2_{\alpha dd} = 36.7 \pm 16.7$ F.

The nuclear vertex constant of the virtual decays $A \to B + C$ are important in the analysis of various nuclear reactions. A theoretical calculation of these constants yields information on the form of the *NN* interaction. So far, a microscopic calculation of the vertex constants was performed only for the simplest two- and three-nucleon systems.[1] In the present paper we present the results of the first calculations of these constants for the decays $\alpha \to t + p$ and $\alpha \to d + d$.

Our calculations are based on solving the integral equations for four nucleons in a model with a separable spin-dependent *NN* potential.[2] The α-particle binding energy in this model is $E_\alpha = -45.73$ MeV, and the tritium binding energy is $E_T = -11.03$ MeV. Since we do not take into account the Coulomb interaction and other effects that violate isotopic invariance, we do not distinguish between the decays $\alpha \to t + p$ and $\alpha \to h + n$, using a single notation $\alpha \to T + N$.

The vertex constants $G^2_{\alpha TN}$ and $G^2_{\alpha dd}$, corresponding to the virtual decays $\alpha \to T + N$ and $\alpha \to d + d$, are expressed in terms of the residues of the *S*-wave amplitudes of elastic *TN* and *dd* scattering in a state with zero spin and isospin, taken at the pole $E = E_\alpha$:

$$G^2_{\alpha TN} = \frac{1}{2} \lim_{E \to E_\alpha} (E - E_\alpha) T_{TN}(E), \quad G^2_{\alpha dd} = \lim_{E \to E_\alpha} (E - E_\alpha) T_{dd}(E), \tag{1}$$

where *E* is the total c.m.s. energy. The normalization of the amplitudes T_{TN} and T_{dd} is as follows:

$$T_{TN} = -\frac{8\pi}{3m} e^{i\delta_T}\frac{\sin\delta_T}{k_T}, \quad T_{dd} = -\frac{2\pi}{m} e^{i\delta_d}\frac{\sin\delta_d}{k_d}, \tag{2}$$

where m is the nucleon mass, $k_T = \sqrt{3m(E+E_T)/2}$, $k_d = \sqrt{2m(E+2E_d)}$, E_T (E_d) is the binding energy of the tritium (deuteron), and $\hbar = c = 1$. The coefficient 1/2 in (1) is due to the isospin kinematics. The quantities $G^2_{\alpha TN}$ and $G^2_{\alpha dd}$ have the dimension of length.

Using the resonant Hilbert-Schmidt expansion method[2] we can easily show that

$$G^2_{\alpha TN} = \frac{8\pi^2}{3\sqrt{3}\gamma_\eta\gamma_e}\,[A_1(i\kappa_T;\,E_\alpha)]^2, \quad \kappa_T^2 = 2m(E_T - E_\alpha); \tag{3}$$

$$G^2_{\alpha dd} = \frac{8\pi^2}{\gamma_\xi\gamma_e}\,[B_1^0(i\kappa_d;\,E_\alpha)]^2, \quad \kappa_d^2 = 2m(2E_d - E_\alpha). \tag{4}$$

In (3) and (4) the quantities $A_1(q;z) = X_1^{-1}(z - q^2/2m)a_1(q;z)$ and $B_1^0(q;z) = Y_{01}^{-1}(z - q^2/2m)b_{01}(q;z)$ have the meaning of α-particle vertex functions,[3] $X_1(z)$ and $Y_{01}(z)$ are the propagation functions of the 3 +1 and 2 +2 resonances, which are expressed in terms of the eigenvalues $\eta_1(z)$ and $\eta_{01}(z)$ of the corresponding integral equations, $e_1(z)$ is the eigenvalue of the four-particle problem ($e_1(E_\alpha) = 1$),

$$\gamma_\eta = \left.\frac{d\eta_1}{dz}\right|_{z=E_T}, \quad \gamma_\xi = \left.\frac{d\xi_1}{dz}\right|_{z=2E_d}, \quad \gamma_e = \left.\frac{de_1}{dz}\right|_{z=E_\alpha}.$$

The vertex functions A_n and B_n^i ($i = 0$ or 1 is the isotopic index corresponding to real or virtual deuterons) satisfy the system of equations

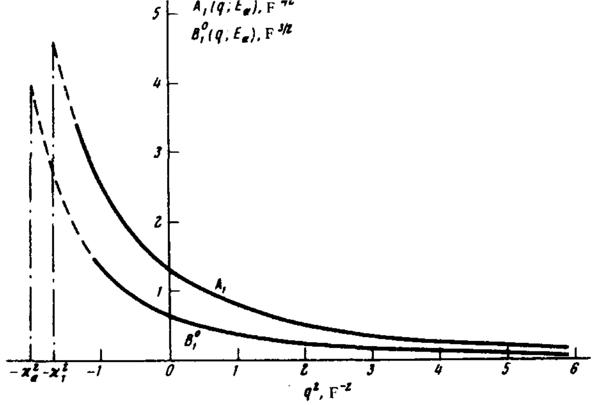
$$A_n(q;E_\alpha) = \sum_{n'}\left\{\int_0^\infty C_{nn'}(q,q';E_\alpha)X_{n'}\left(E_\alpha - \frac{q'^2}{2m}\right)A_{n'}(q';E_\alpha)q'^2dq' + \sum_{i=0}^{1}\int_0^\infty D^i_{nn'}(q,q';E_\alpha)Y_{in'}\left(E_\alpha - \frac{q'^2}{2m}\right)B^i_{n'}(q';E_\alpha)q'^2dq'\right\}, \tag{5}$$

$$B^i_n(q;E_\alpha) = \sum_{n'}\int D^i_{n'n}(q',q;E_\alpha)X_{n'}\left(E_\alpha - \frac{q'^2}{2m}\right)A_{n'}(q',E_\alpha)q'^2dq', \tag{6}$$

where the "potentials" $C_{nn'}$ and $D_{nn'}$ are defined in[2]. The normalization of these functions is

$$\sum_n\int_0^\infty\left\{A_n^2(q;E_\alpha)X_n\left(E_\alpha - \frac{q^2}{2m}\right) + \sum_{i=0}^{1} B_n^{i\,2}(q;E_\alpha)Y_{in}\left(E_\alpha - \frac{q^2}{2m}\right)\right\}q^2dq = 1. \tag{7}$$

The system (5) and (6) was solved numerically with account taken of three functions A_n and two functions B_n^i (for each value $i = 0$ and 1). This cutoff of the system of equations ensures sufficient accuracy in the calculation of the binding energy of the α particle.[2] Further, using the values of the functions $A_1(q;E_\alpha)$ and $B_1^0(q;E_\alpha)$ at $q^2 > 0$ (we recall that A_n and B_n^i are even functions of the momentum), we can determine in principle the form factors necessary for the calculation of the vertex constants at the points $\kappa_T^2 = 1.67$ F^{-2} and $\kappa_d^2 = 1.99$ F^{-2}, by calculating the integrals in the right-hand sides of (5) and (6). However, in view of the resultant singularities of the integrands, a direct calculation of the integrals in (6) at $q^2 \geq 1.1$ F^{-2}. To find the quantities $A_1(i\kappa_T;E_\alpha)$ and $B_1^0(i\kappa_d;E_\alpha)$ we therefore use an analytic approximation of the vertex functions by means of polynomials:



Vertex functions of α particle.

$$A_1(q;E_\alpha) = \sum_{k=0}^{N_1} a_k q^{2k}, \quad B_1^0(q;E_\alpha) = \sum_{k=0}^{N_2} b_k q^{2k} \tag{8}$$

with subsequent extrapolation to the points $i\kappa_T$ and $i\kappa_d$. The analytic approximation was carried out at different values of N_1 and N_2 ($3 \leq N_1,\ N_2 \leq 9$). We used also different regions of the values of q^2 to determine the coefficients a_k and b_k from the calculated values of $A_1(q;E_\alpha)$ and $B_1^0(q;E_\alpha)$. For the function $A_1(q;E_\alpha)$, various approximation methods that describe well the region $q^2 \geq -1.3$ F^{-2} lead to close values of $A_1(i\chi_T;E_\alpha)$, at 4.64–4.87 F$^{3/2}$. Substituting these values in (3), we obtain

$$G^2_{\alpha TN} = 17.5 \pm 0.9 \text{ F}.$$

In the case of $B_1^0(q;E_\alpha)$, the scatter due to the ambiguity of the approximation is large (apparently owing to the large extrapolation interval), $B_1^0(i\kappa_d;E_\alpha) = 3.95$–6.46 F$^{3/2}$, from which we get in accordance with (4)

$$G^2_{\alpha dd} = 36.7 \pm 16.7 \text{ F}.$$

The calculated values of the vertex functions are shown in the figure. The solid sections of the curves were obtained by solving the system (5) and (6) at $q^2 > 0$ or by calculating the integrals in (5) and (6) at $q^2 < 0$. The dashed sections are plotted by extrapolation in accordance with formulas (8) at $N_1 = 9$ and $N_2 = 6$.

In view of the absence of other microscopic calculations of the vertex constants $G^2_{\alpha TN}$ and $G^2_{\alpha dd}$, we can compare our results only with the phenomenological values extracted from data on nuclear reactions. The values of $G^2_{\alpha TN}$ obtained by analyzing different reactions within the framework of the peripheral model,[5–7] the dispersion relation,[8–10] and the K-matrix approach[11,12] vary in the interval 7—13 F. There are very few data on the constant $G^2_{\alpha dd}$; an analysis of the (d, α) reactions and of αd scattering with the aid of the peripheral model yields $G^2_{\alpha dd} = 12—30$ F. The slight excess of the values obtained by us for the vertex constants over the phenomenological values may be due to the noticeable overbinding of the α particle for the employed model of the NN potential.

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2) The Hilbert-Schmidt method as applied to the four-nucleon problem was considered in[3]. For the calculation of three-particle vertex constants with the aid of this method see[4].
3) We use here and below the notation of[2].

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