Connection between the decay probabilities of two-phonon states and quadrupole moments of phonons in atomic nuclei

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We analyze the connection between the quadrupole moments of the single-phonon states and the probabilities of two-phonon annihilation.

Experimental information on anharmonic effects in atomic nuclei is based mainly on measurements of the quadrupole moments of single-phonon states and of the decay probabilities of two-phonon excitations. In classical hydrodynamics, anharmonic effects are universal: the multipole-moment operator $\mathbf{M}_{\lambda\lambda}=\int r_\lambda Y_{\lambda\lambda}(\mathbf{n})\,\rho(r)\,d^3r$ is expressed directly in terms of the operators d_{LM} of the drop-surface deformation $\delta R=R\Sigma d_{LM}Y_{LM}$. We have[1]

$$\rho(\mathbf{r},R)=\rho_0+\frac{\partial\rho}{\partial R}\delta R+\frac{1}{2}\frac{\partial^2\rho}{\partial R^2}\delta R^2+\ldots, \quad (1)$$

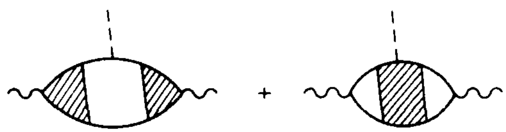
Then

$$\mathbf{M}\lambda\lambda=\frac{3ZR}{4\pi}\left\{d_{\lambda\lambda}-\frac{\lambda+2}{2}\Sigma d_{L_1M_1}d_{L_2M_2}\int Y_{L_1M_1}Y_{L_2M_2}Y_{\lambda\lambda}d\mathbf{n}\right\}. \quad (2)$$

We see from this that the quadrupole moments Q and the probabilities $W_{2f\to 0}$ of two-phonon annihilation are rigidly interrelated. However, experiment does not confirm this: whereas the values of $W_{2f\to 0}$ turn out to be of the order of the single-particle values, the quadrupole moments Q greatly exceed the single-particle estimates. The present article is devoted to an analysis of this contradiction.

Before we write out the concrete formulas, let us explain the results of the subsequent analysis. It turns out that in a quantum liquid drop the operator $\mathbf{M}_{\lambda\lambda}$ has, in addition to the classical hydrodynamic component (2), also typical quantum components proportional not to the external field $r^\lambda Y_{\lambda\lambda}$, but to the effective field $V(\mathbf{r},\omega)$. The value of $V(\mathbf{r},\omega)$ depends essentially on the frequency ω of the external field. V has a pole at the phonon frequency $\omega=\omega_L$. Inasmuch as the quadrupole moment is a static characteristic of a state, and in two-phonon annihilation the frequency is $\omega\approx 2\omega_L$, matrix elements of the effective field enter at different values of ω, so that the relations between $W_{2f\to 0}$ and Q will be much more complicated than in the hydrodynamic case.

The set of diagrams for the matrix element to two-phonon annihilation is shown in the figure. The solid lines correspond to the propagation functions of the particles, the shaded triangles correspond to the blocks g of phonon decay into a particle-hole pair,[2] and the square corresponds to total amplitude of the two-particle interaction in the external field V^0. The corresponding analytic expression for Q can be obtained with the aid of the method proposed in[3]



$$Q = (g(\mathbf{r}_1)T(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)V^\circ(\mathbf{r}_2, \omega)g(\mathbf{r}_3)) + (V^\circ(\mathbf{r}_2, \omega)\, G(\mathbf{r}_2\mathbf{r}_3)G(\mathbf{r}_2\mathbf{r}_1)M(\mathbf{r}_1, \mathbf{r}_3)). \tag{3}$$

The outer parentheses in (3) denote integration with respect to the coordinates $\mathbf{r}$. $T(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$ represents the change of the density of the noninteracting quasiparticles at the point $\mathbf{r}_2$ when unity external fields act at the points $\mathbf{r}_1$ and $\mathbf{r}_3$. M is the aggregate of all the "emcompassing" diagrams for the scattering of a phonon by a particle (we recall that it is necessary to put $\omega = 0$ in the calculation of Q and $\omega = 2\omega_L$ in the determination of $W_{2f\to 0}$). The equation for the amplitude M is obtained as in the ordinary Compton effect by variation of the equation for the amplitude g[2]

$$g(\omega_L) = FA(\omega_L)\, g(\omega_L). \tag{4}$$

Here F is the local quasiparticle-scattering amplitude.[2] $A(\omega, r_1, r_2)$ is the standard propagator of the theory of finite Fermi systems, which gives the change of the density of the noninteracting quasiparticles at the point r_1 when a unit field is applied to the system at the point r_2. Then

$$M = \delta FAg + F\delta Ag + FAM. \tag{5}$$

Here δA is the change of the propagator and δF is the change of the local amplitude of the interaction as a result of phonon excitation. It is usually assumed that $\delta F = 0$, since it is assumed that the field produced by the low-lying excitation is long-wave. In fact this is not so: the phonon block g has a sharp peak on the surface (the phonons are surface excitations to a considerable degree)[4] and the phonon field is thus concentrated principally on the surface of the system. In each small section of space, this field is practically the same as the field produced when the center of gravity of the system is shifted ($g(\mathbf{r}) \sim \partial U/\partial \mathbf{r}$ in the zero-order approximation). However, the shift produces the same increment for all the coordinates on which F depends, and as a result it turns out that

$$\delta F(\mathbf{r}, \mathbf{r}') = \beta_L\left(\frac{\partial F(\mathbf{r}, \mathbf{r}')}{\partial \mathbf{r}} + \frac{\partial F(\mathbf{r}, \mathbf{r}')}{\partial \mathbf{r}'}\right)Y_{LM}$$

(L is the angular momentum of the phonon).

The change δF is indeed equal to zero in the entire volume. It exists only on the edge of the drop. This term cannot be neglected, however. It is precisely this term which is responsible for the appearance of the classical hydrodynamic anharmonicity (2). To calculate (5) it is necessary to know also the change of A in the field of the phonon. The graphical result is $\delta A = (Tg)$. Substituting now the symbolic solution (5)

$$M = (1 - FA)^{-1}(\delta FAg + FgTg) \tag{6}$$

in formula (3) for Q, we obtain after simple transformations

$$Q = \left(d_{LM} V^\circ Y_{LM} \frac{\partial}{\partial r} Ag\right) + (Vg_{LM}\, Tg_{LM}) + d_{LM}\left(Y_{LM} VA \frac{\partial g}{\partial r}\right) - d_{LM}\left(Y_{LM} V \frac{\partial}{\partial r} Ag_{LM}\right). \tag{7}$$

We note one important feature of (7): in the case of pure shift $L = 1$, $g_{1M} = d_{1M}(\partial\Sigma/\partial r)$,[4] all the terms with the effective field cancel each other rigorously and only the hydrodynamic component is left.

An appreciable cancellation should take place also when $L \neq 1$. However, in spite of this, the quantum component with $V(\mathbf{r})$ plays the decisive role in (7).

The point is that $V(\mathbf{r})$ is much larger than $V^0(\mathbf{r})$ on the surface of the system. The reason for this enhancement is that in an external field the edge of the drop becomes deformed (as is seen even from formula (1)), and this leads to a strong enhancement of V. We can write the rough estimate

$$V_\lambda(r = R) \sim \frac{1}{C_\lambda - \omega^2 B_\lambda}\left(\frac{\partial \rho}{\partial r} V^\circ\right),$$

where C_λ is the quantum rigidity and B_λ is the mass coefficient. As shown in[4], the rigidity C_λ is much less than the single-particle value and it is this smallness which serves as a measure of the smallness of V^0/V. It appears that the estimated surface part of the entire integral (7) is of the order of the optimal term of the sum, i.e., $\sim \partial^2\rho/\partial r^2$. It follows then that quantum component of Q is larger than the classical component in the ratio V/V_0. It is interesting that $V(\omega = 2\omega_L)/V(\omega = 0) \approx -1/3$.

If it is assumed that the quantum component of Q predominates, then this leads, first to an important sign rule, namely the signs of the matrix element $BE_2^+ \to 0$ and $Q(2^+)$ are opposite! Second, we again obtain a rigid connection between $BE2_2^+ \to 0$ and $Q(2^+)$, but the coefficient is $\sim 1/3$ of the hydrodynamic value. This decrease of the probability of two-photon annihilation in comparison with the quadrupole moments of the phonons agrees with experiment. Of course, to obtain exact relations between $BE2_2^+ \to 0$ and $Q(2^+)$ it is necessary to perform numerical calculations for each concrete case, using formula (7).

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