Stratification of ionospheric plasma in the region of reflection of the ordinary radio wave

V. V. Vas'kov and A. V. Gurevich

P. N. Lebedev Physics Institute, USSR Academy of Sciences
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We consider the mechanism of the instability that leads to stratification of the ionospheric plasma in the region of reflection of a high-power radio wave.

Recently Utlaut, Cohen, *et al.*[1] and Getmantsev *et al.*[2] have observed a new nonlinear effect of self-action and interaction of radio waves in the upper ionosphere, namely that the intensity of a strong ordinary wave reflected from the F layer decreases rather than increases with increasing radiation power; radio waves of other frequencies, reflected from the same region as the perturbing wave, are also substantially attenuated. At the same time, a sharp increase is observed in the large-scale ionospheric inhomogeneities.[3,4] The interaction of the radio waves with these inhomogeneities (scattering, linear wave transformation) seem to be the cause of this nonlinear effect. The purpose of the present paper is a theoretical investigation of these phenomena. It is shown that an instability of the self-focusing type is produced in the region of reflection of a high-power ordinary wave in the ionosphere, and leads to an intensive growth of the large-scale inhomogeneities. This instability is a result of the change produced in the concentration by plasma diffusion from the region heated by the wave.

It is important that under the conditions of the F layer the dependence of the concentration N on the intensity of the perturbing wave is essentially nonlocal both in space and in time. This determines the character of the development of the instability. We note also that parametric excitation of Langmuir oscillations leads to an additional heating (in comparison with ohmic heating) of the electrons, and consequently greatly strengthens the effect.

The complete system of equations describing the interaction of the ionospheric plasma with an ordinary wave of frequency ω normally incident on a plane-layered ionosphere takes, in the region of the reflection point where $\epsilon_0(z=0)=0$, the form

$$\left\{\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial x^2}+\frac{\omega^2}{c^2}\left[\epsilon_0(z)-\frac{\Delta N}{N}\right]\right\}E=0,$$
$$\frac{\Delta N}{N}=\int_0^t dt_1\int dz_1\,\mathrm{tg}\,\alpha\, W G\left[(z-z_1)\mathrm{tg}\,\alpha,\ t-t_1\right]. \qquad (1)$$

Here E is the amplitude of the electric field of the wave, $\epsilon_0=(1-4\pi e^2N/m\omega^2)$ is the dielectric constant of the unperturbed plasma, and $\Delta N/N$ is the relative perturbation of the electron concentration, which is determined by the power W dissipated per unit volume.

The perturbation $\Delta N/N$ is determined by solving the system of equations describing the thermal conductivity and the diffusion of the plasma along the magnetic-field force line.[5] G is the Green's function of this system. Further, α is the angle between the earth's magnetic field $\mathbf{H}$ and the vertical, and t is the time from the instant when the field is turned on. We use a coordinate system with the x axis perpendicular to the wave-propagation plane; the coordinate z is reckoned from the reflection plane $z=0$ along the group-velocity vector ($z\perp\mathbf{H}$).

The dissipated power W which enters in (1) is expressed in terms of the wave energy density ($\nu_e\ll\omega$)

$$W=\frac{|E|^2}{8\pi}\nu_e\begin{cases}2\dfrac{|E|^2}{E_{\rm thr}^2}-1 & |E|^2>E_{\rm thr}^2\\[2mm] 1 & |E|^2<E_{\rm thr}^2\end{cases}, \qquad (2)$$

where ν_e is the electron collision frequency and $E^2_{\rm thr}=16\pi NT_iF\nu_e/\omega$ is the threshold for the excitation of the parametric instability[6] ($F\approx1.75$).

The Green's function $G(x,t)$, in a wide range of the variables ($t<\tau_T D_T/D_a$, τ_N and $x^2<4D_T\tau_T$), is equal to[7]

$$G(x,t)=-\left(\frac{3}{2}NT_e\right)^{-1}\frac{k_T}{\sqrt{\pi}}\frac{D_a}{D_T-D_a}\frac{1}{x}\left(u_1e^{-u_1^2}-u_2e^{-u_2^2}\right),$$
$$u_1=x/\sqrt{4D_at},\quad u_2=x/\sqrt{4D_Tt}. \qquad (3)$$

Here D_a and $D_T=\kappa_e/N$ are the coefficients of the ambipolar diffusion and of the electronic thermal conductivity, $\tau_T=1/\nu_e\delta\phi_T$ is the time of establishment of the electron temperature ($\phi_T\sim1$ is the non-isothermy factor), τ_N is the electron lifetime, and $k_T\sim1$ is the thermal-diffusion ratio.[5]

Within the framework of applicability of geometrical optics, the unperturbed wave equation (1) has the solution

$$E=\epsilon_0^{-1/4}\sin\alpha^{-1/2}\{E_0e^{-i\phi}+E_0e^{i\phi}\},$$
$$\phi=\int_0^z k_0(z_1)dz_1-\frac{\pi}{4},\qquad k_0=\frac{\omega}{c}\sqrt{\epsilon_0}, \qquad (4)$$

where in the case of weak nonlinear absorption of the RF wave (at $E\lesssim(2-3)E_{\rm thr}$) the amplitudes of the incident and reflected waves are equal to the amplitude E_0 of the wave incident on the ionosphere.

Let us examine the stability of this solution. We assume the incident wave to be homogeneous in the direction of the x axis and orthogonal to its propagation plane. Then

$$E_{\rm inc} = E_0 + E_1 e^{ikx} + E_2 e^{-ikx}; \quad E_{\rm ref} = E_0 + E_3 e^{ikx} + E_4 e^{-ikx}. \tag{5}$$

We linearize Eqs. (1)—(3) and take the Laplace transforms of the corresponding quantities in the form

$$a(\gamma) = \int_0^\infty a(t) e^{-\gamma t} dt.$$

As a result we obtain a system of equations for the dimensionless amplitudes $a_{1,3} = E_{1,3}/E_0$ and $a_{2,4} = E^*_{2,4}/E^*_0$:

$$-i\frac{da_1}{dz} - \frac{k^2}{2k_0}a_1 - \frac{\omega^2}{c^2 2k_0}\frac{\delta\Delta N}{N} = 0,$$

$$i\frac{da_2}{dz} - \frac{k^2}{2k_0}a_2 - \frac{\omega^2}{c^2 2k_0}\frac{\delta\Delta N}{N} = 0, \tag{6}$$

$$a_1 - a_4 = A\exp\left\{i\int_0^z \frac{k^2}{2k_0}dz_1\right\}, \quad a_2 - a_3 = -A\exp\left\{-i\int_0^z \frac{k^2}{2k_0}dz_1\right\},$$

where the concentration perturbation $\delta\Delta N/N$ is described by the expression

$$\frac{\delta\Delta N}{N} = -\frac{k_T}{2\sqrt{\gamma}}\frac{D_a}{D_T - D_a}\int dz_1 \operatorname{tg}\alpha\left(\frac{3}{2}NT_e\right)^{-1}\delta W$$
$$\times\left[D_a^{-1/2}\exp\left\{-\sqrt{\frac{\gamma}{D_a}}\,|z - z_1|\operatorname{tg}\alpha\right\} - D_T^{-1/2}\exp\left\{-\sqrt{\frac{\gamma}{D_T}}\,|z - z_1|\operatorname{tg}\alpha\right\}\right],$$

$$\delta W = \frac{3}{2}NT_e\tau_T^{-1}\frac{1}{2}L(a_1 + a_2 + a_3 + a_4). \tag{7}$$

$$L = \frac{\overline{|E|^2}}{E_p^2\phi_T}\left\{1 + \left[C_1\frac{\overline{|E|^2}}{E_{\rm th}^2} - 2\right]\theta(2\overline{|E|^2} - 1\right\}, \quad \theta(x) = \begin{cases}1 & x > 0\\ 0 & x < 0\end{cases}.$$

Here $E_p^2 = 3mT_e\delta\omega^2/e^2$ is the characteristic plasma field,[5] $\overline{|E|^2} = 2|E_0|^2/\sqrt{\epsilon_0}\sin\alpha$ is the unperturbed-wave intensity averaged over the period, and $C_1 = 6$ in the limit when $\overline{|E|^2} \gg E^2_{\rm thr}$.

The solutions of the linear equations (6) and (7) were obtained in different limiting cases in the region of large and small values of k. An analysis of these solutions shows that small perturbations of the incident-wave field increase exponentially with time. As a result, inhomogeneities of the electron concentration are produced and grow in the wave-reflection region. In the direction perpendicular to the wave-propagation plane, the characteristic dimension of these inhomogeneities, $\lambda_\perp \approx 2\pi(8q\beta)^{-1/6}k_m^{-1}$ (see below[8]), is of the order of the maximum length of the incident wave near its reflection point: $\lambda_\perp \sim 2\pi/k_m$, $k_m = \omega/cR$, $R = (\omega/c\mu\sin\alpha)^{1/3}$, and $\mu = |\nabla N/N|$. In the F-layer of the ionosphere, the dimension of the produced inhomogeneities transverse to the magnetic field is $\lambda_\perp \sim 1$ km. The longitudinal dimension is $\lambda_\parallel \sim \sqrt{D_{ar}} \sim 10$ km. It is determined by the longitudinal diffusion of the plasma during the characteristic time τ of the inhomogeneity growth. The time $\tau = 1/\gamma_m$, where γ_m is the maximum instability increment, depends on the pump-wave intensity

$$\tau = \tau_T\frac{D_T}{D_a}\left[1 + \left(\frac{\beta q}{2}\right)^{2/3}k_m z_0\right]\left(\frac{8q}{\beta^2}\right)^{2/3}, \tag{8}$$

$$q = \operatorname{tg}\alpha/k_m\sqrt{4D_T\tau_T}, \quad \beta = \frac{R^2 k_T}{\sqrt{D_T\tau_T}}\int L\,dz\operatorname{tg}\alpha.$$

The quantity L is defined in (7), and the parameter z_0 denotes the lower limit of the region of effective plasma heating, $0 < z < z_0$.

In the F layer of the ionosphere, at low powers of the perturbing waves, we have $\tau \sim \tau_T \approx 1/\nu_e\delta \sim 20$—40 sec; with increasing power, and especially when the threshold of excitation of the parametric instability is exceeded, the time τ decreases rapidly (see[7,8]). Thus, at $E \gtrsim 2E_{\rm thr}$ it already amounts to 2—5 sec. All this agrees with the results of the observations.[1-4]

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