Dynamic turbulence regimes of plasma oscillations

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It is shown analytically and with the aid of computer calculations that stabilization of parametric instability of Langmuir oscillations in strong high-frequency fields can be produced by generating in-phase spatial harmonics of the field of the unstable oscillations.

In connection with the problem of condensation of Langmuir waves in the region of small wave vectors k, the possible regimes of strong Langmuir turbulence have been widely discussed of late[1–4]; these regimes are realized both in the case of a sufficiently large ratio of the wave energy density W to the plasma thermal-energy density

$$\frac{W}{NT_e} > (kr_D)^2 . \qquad (1)$$

Here N, T_e, and r_D are the density, temperature, and the Debye radius of the electrons. If the inequality (1) is satisfied, it becomes possible for the Langmuir-turbulence energy to become nonlinearly dissipated into small scales.

Two limiting strong-turbulence regimes can be considered. In the first the turbulence is a gas of weakly interacting Langmuir solitons (a development of this idea was undertaken in[4]), and in the second the turbulence is "dynamic." In the latter case, which we investigate below, the redistribution of the energy over the spectrum is due to the interaction of the phased spatial harmonics of the field. The system of equations describing such an interaction consists of a parabolic equation for the plasma-oscillation field amplitude $E(x, t)\exp(i\omega_p t)$ and the wave equation for small plasma-turbulence perturbations.

In the one-dimensional case this system, at a specified homogeneous external electric field $E = E_0 \exp(i\omega_0 t)$ in the plasma, takes the form

$$-2i\frac{\partial A_1}{\partial t} + \frac{\partial^2 A_1}{\partial x^2} - nA_1 = nA_0 \exp\{i\Omega t\} ,$$
$$\frac{1}{u^2}\frac{\partial^2 n}{\partial t^2} - \Gamma\frac{\partial^2 n}{\partial x^2} = \frac{\partial^2 |A_1|^2}{\partial x^2} + A_0\frac{\partial^2}{\partial x^2}\{A_1^* e^{i\Omega t} + \text{c.c.}\}. \qquad (2)$$

We have introduced here the dimensionless variables $x_M = x(3r_D\sqrt{g})^{-1}$; $t_M = \omega_p t/3g$; $A_{1,0} = E_{1,0}(16\pi NT_e/3g)^{-1/2}$; $n = 3g(\delta n/N_e)$; $g = [T_e/(T_e + T_i)](M/m)u^2$, T_e and T_i are the temperatures of the electrons and of the ions, u is the scaling parameter (the dimensionless sound velocity), Γ is a coefficient introduced for convenience in the calculation ($\Gamma = 0.1$), $A_0 = \text{const}$, and $\Omega = \omega_0 - \omega_p$. The system (2), generally speaking, is not conservative, with the exception of the case of zero detuning $\Omega = 0$, and describes both modified decays and the modulation instability in the plasma. Equations (2) with $\Omega = 0$ correspond also to the initial-condition problems in the absence of an external source, if A_1 is taken to mean the amplitude of the plasma oscillations after subtracting the homogeneous field components A_0, which is specified at the initial instant of time. The system (2) was solved with a computer under the periodic boundary conditions

$$E(x=-l) = E(x=l); \quad \frac{\partial E}{\partial x}(x=-l) = \frac{\partial E}{\partial x}(x=l);$$
$$n(x=-l) = n(x=l); \quad \frac{\partial n}{\partial x}(x=-l) = \frac{\partial n}{\partial x}(x=l). \qquad (3)$$

The numerical experiments were performed with a rather large set of values of the parameters and initial conditions of the problem. Let us discuss, in particular, the solution of the problem with initial conditions in the absence of an external source ($\Omega = 0$). The initial field was represented in the form of a sum of a homogeneous high-frequency field of relatively large amplitude A_0 and spatially-periodic multiple harmonics of low energy and with arbitrary initial phases. Typical results are shown in Figs. 1–3. As seen from Fig. 2, phasing of the interaction and an exponential growth of the harmonic, corresponding to the linear stage of the instability, takes place at first. The growth then gives way to a quasiperiodic strongly nonlinear regime. At the same initial amplitude of the homogeneous field, the time behavior of the total energy density (Fig. 2) of the plasma waves depends little on the concrete form of the initial conditions. During the phase of saturation in the dynamic regime, there is formed a universal spectrum, averaged over the time, of the spatial scales of the plasma waves, $|A_k|^2 \sim k^{-2}$ (Fig. 3), with a sharp cutoff at a certain k_{max}, in analogy with the results of the numerical experiment.[3]

Important characteristics of the solution for the case of the initial problem[1)] can be found also analytically, by using the integrals of motion of the system (2) ($\Omega = 0$):

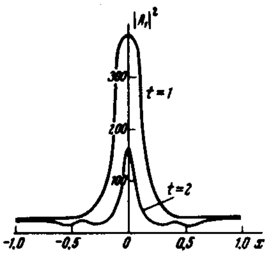

FIG. 1.

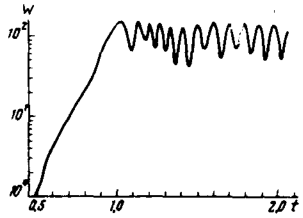

FIG. 2

$$J_1 = \int_{-l}^{l} |A_1 + A_0|^2 \, dx \,, \tag{4}$$

$$J = \int_{-l}^{l} \left\{ \left| \frac{\partial A}{\partial x} \right|^2 + n \left| A_1 + A_0 \right|^2 + \frac{n^2}{2} + \frac{v^2}{2} \right\} dx \,, \qquad \frac{\partial n}{\partial t} + \frac{\partial v}{\partial x} = 0. \tag{5}$$

The first integral makes it possible to obtain the upper bound of the average energy density of the perturbation ($\overline{W} = (1/2l)\int_{-l}^{l} |A_1|^2 dx$)

$$\overline{W} \lesssim 4A_0^2. \tag{6}$$

From the second equation we can obtain a rigorous estimate of the maximum amplitude of the oscillation field

$$|A_1|^2_{max} \lesssim 2A_0^4 \, l^2 \left[1 + 0\left(\frac{1}{A_0^2 l} \right) \right]. \tag{7}$$

From (6) and (7) we get the value of the maximum wave number in the spectrum of the Langmuir waves

$$\frac{k_{max}}{k_0} = \frac{l}{l_{min}} \sim \frac{A_0^2 l^2}{2} \,. \tag{8}$$

The results of the numerical calculation agree in order of magnitude with the estimates (6)—(8) (see Fig. 2).

In the case when the energy J_1 of the total field is not conserved, for example in the case of parametric instability in the field of an external wave with $\Omega \neq 0$, the estimates (6) and (7) are incorrect. However, the numerical calculations (in the final time interval $t \sim 6u^2(M/m)(1/\omega_p)T_e/(T_e + T_i)$ indicate that here, too, the instability development proceeds analogously, i.e., the source is "disconnected," as it were, during a certain stage, and subsequently the distribution of the field and the energy density of the waves oscillate about the mean values.

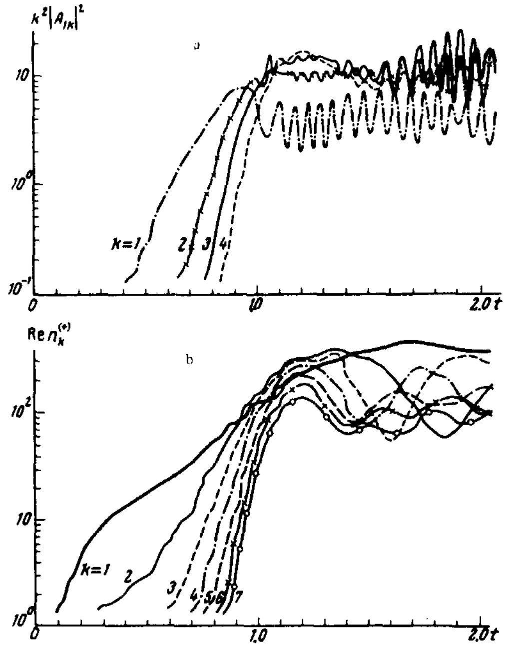

FIG. 3.

We note in conclusion that in the case when the generation of phased spatial harmonics plays an essential role, the electron velocity distribution function should contain, under certain conditions, groups of fast particles with multiple velocities. Such particle groups were observed in experiments on parametric instability[5] and can serve as indirect evidence of the correctness of the considered dynamic model.

[1]) This includes the case with nonzero pump $A_0 \neq 0$ at zero detuning $\Omega = 0$ in (2).

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