High-frequency behavior of the amplitude of a nonstationary Josephson current

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The spectrum of the elementary excitations in a superconductor is determined in a frequency region not confined to the Debye region. The spectrum is used to calculate the spectral characteristics of the Josephson tunnel current.

A detailed theoretical description of the nonstationary Josephson effect at a voltage exceeding the total energy gap of the superconductors in contact is presented in[1,2]. Experimental observation of the effect was reported in[3-6].

In this study, using the electron-phonon interaction model, we have obtained a steeper decrease of the Josephson-current amplitude at frequencies above the limiting frequency of the phonon spectrum in the metal.

The spectral characteristic ("amplitude"[1]) of the Josephson tunnel current between superconductors 1 and 2 is given by[2]

$$j(\omega) = \frac{1}{2\pi^2 eR} \sum_{i \neq j = 1}^{2} \int_{-\infty}^{\infty} d\omega' \,\mathrm{th}\frac{\omega'}{2kT} F_i^R(\omega' + \omega)\,\mathrm{Im}\, F_j^R(\omega'), \quad (1)$$

where

$$F(\omega) = \frac{1}{4\pi m p_F} \int d\mathbf{p}\, F(\mathbf{p}, \omega), \quad F^R(\mathbf{p}, \omega) = \begin{cases} F(\mathbf{p}, \omega), & \omega > 0 \\ F^*(\mathbf{p}, \omega), & \omega < 0 \end{cases}$$

$F(\mathbf{p}, \omega)$ has the same form as the Fourier component of the Gor'kov function at absolute zero temperature,[7], but with $\Delta_0(T)$ (see below), R is the resistance of the Josephson junction in the normal state, and p_F is the electron momentum at the Fermi level.

For $F(\mathbf{p}, \omega)$ of an isolated pure superconductor at absolute zero, the equations of[7] hold true in first-order approximation in the vertex part. We obtain an approximate solution of the equations for $\overline{F}_\omega$ and $\overline{G}_\omega$ with the Green's function of the noninteracting phonons $D^{(0)}(\mathbf{k}, \omega) = u^2\mathbf{k}^2[\omega^2 - (u|\mathbf{k}| - i\delta)^2]^{-1}$. After integrating with respect to the angles between the electron and phonon momenta $\mathbf{p}$ and $\mathbf{k}$, the expression for $\overline{F}_\omega$ becomes

$$\overline{F}_\omega = -\frac{ig^2}{(2\pi\hbar)^4} \int_{-\infty}^{\infty} d\omega' \int_0^{k_D} 2\pi k^2 dk \frac{m}{pk} \frac{u^2k^2}{(\omega' - \omega)^2 - (uk - i\delta)^2} \frac{\overline{F}_{\omega'}}{f(\omega')}$$
$$\times \left\{ \mathrm{Ar\,th} \frac{[(p+k)^2 - p_F^2](2m)^{-1} + ig^2 \overline{G}''_{-\omega'}}{f(\omega')} - \omega' \,\mathrm{Ar\,th} \frac{[(p-k)^2 - p_F^2](2m)^{-1} + ig^2 \overline{G}''_{\omega'}}{f(\omega')} \right\},$$

where g is the electron-phonon interaction constant, u is the speed of sound in the metal, k_D is the limiting value of the phonon momentum in the crystal lattice, $\xi = (\mathbf{p}^2 - p_F^2)/2m$, $f(\omega) = [(\omega + ig^2\overline{G}'_{\omega})^2 - g^4|\overline{F}_\omega|^2]^{1/2}$, and $\overline{G}'_\omega$ and $\overline{G}''_\omega$ are the Green's-function components odd and even in ω. The integrand has a discontinuity in the complex plane on the cut from $-\infty$ to $-\Delta_0$ and from Δ_0 to ∞, with a value of $\{\}$ equal to $(-i\pi)$ everywhere except in the interval

$$(p + p_F - k)|p - p_F - k| \lessgtr 2mf(\omega') \lessgtr (p + p_F + k)(p - p_F + k),$$

where $\{\}$ is equal to $(-i\pi/2)$. For the momenta $|p - p_F| < k_D$ and for a total electron energy relative to the Fermi level $\omega < p_F^2/2m$, the contribution from this interval, which is determined by the region of large $k \approx k_D$, is small, $\sim |p - p_F| u^2 m^2 p_F^{-3}$. The remaining expression for $\overline{F}_\omega$ does not depend on $\overline{G}''_\omega$ or on p, and is an even function of ω, while $\overline{G}_\omega$ is correspondingly an odd function of ω.

In first-order approximation, assuming

$$\Delta(\omega) = g^2|\overline{F}_\omega|(1 - ig^2\overline{G}_\omega/\omega)^{-1} = \begin{cases} \Delta_0, & \omega < a\omega_D, \quad a = 1 \\ 0, & \omega > a\omega_D \end{cases} \quad (2)$$

we write down an approximating expression for the solution of the integral equations for $\overline{F}_\omega$ and $\overline{G}_\omega$:

$$ig^2\overline{F}_\omega = -\frac{\mathfrak{z}\Delta_0}{2\omega_D^2}\left[\omega_D^2 + \omega^2\left(\mathrm{Ar\,ch}\, 2\frac{|\omega_D^2 - \omega^2|}{\Delta_0^2} - 2\,\mathrm{Ar\,ch}\frac{\omega}{\Delta_p}\right)\right.$$
$$\left. \times \,\mathrm{Ar\,ch}\, 2\frac{\omega_D^2[(0,6\omega_D)^2 - \omega^2 - i\delta}{\Delta_0^2[10,6\omega_D)^2 + \omega^2]}\right],$$

$$ig^2\overline{G}_\omega = -\frac{\mathfrak{z}}{120\omega_D}\left\{\left[(a\omega_D)^2 - \omega^2\right]\left[\mathrm{Ar\,ch}\frac{(\omega + \omega_D)^2}{\Delta_0(\omega + a\omega_0)}\right.\right.$$
$$\left.\left. - \,\mathrm{Ar\,ch}\frac{(\omega - \omega_D)^2}{\Delta_0(\omega - 2\omega_D - i\delta)}\right] a^2(\omega_D^2 - \omega^2)\ln\frac{\omega + \omega_D}{\omega - \omega_D - i\delta} + 30\omega_D\omega\right\}, \quad (3)$$

where $a^3 + 4a^2 + 1 = 0$ $(a \approx 4)$,

$$\mathfrak{z} = \frac{g^2 m p_F^2}{\hbar^4 2\pi^2 p} \approx \ln^{-1}\frac{2\omega_D}{\Delta_0}, \quad \omega_D \approx uk_D.$$

At $\omega > \omega_D$, the value of $\Delta(\omega)$ decreases like $\sim(\omega_D/\omega)^2$ and the assumed approximation is satisfied accurate to ζ. Accordingly, at $\omega > \omega_D$, the amplitude of the Josephson current decreases

$$j(\omega) \approx \left(1 - i\zeta\frac{\pi}{2}\right)\frac{\hbar\pi\Delta_{01}\Delta_{02}\omega_D^2}{eR\omega^3}, \quad (4)$$

where Δ_0 is the root of the equation $\omega = \Delta(\omega)$. $\Delta_0 = \Delta(0)$ and at finite temperatures $\Delta_0(T)$ has a well-known form.[7] The obtained relation yields a steeper decrease for $\omega > \omega_D$, namely in inverse proportion to the frequency cubed.

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