# Deformation of electromagnetic pulse upon development of parametric instability in a plasma layer

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We consider numerically and analytically a modified decay instability in a bounded plasma layer. We show that deep amplitude modulation of the incident was taken place at the exit from the plasma layer, as well as a lenthening of the electromagnetic pulse due to the "emission" of the turbulence accumulated in the plasma after the pump wave is turned off.

In connection with recent ionospheric experiments, [1–4] interest has increased in the investigation of self-action of an electromagnetic pulse in a plasma layer following development of parametric instability. Contributing to this interest was the observation of anomalous attenuation of the amplitude modulation and lengthening[1–3] of the electromagnetic pulses upon reflection from the $F$ layer of the ionosphere.

A possible explanation of a number of the effects indicated above was proposed in[4,5]. In the present article, in contrast to[4,5], we consider the case of sufficiently strong fields

$$\frac{W}{NT_e} > \left(\frac{m}{M}\right)^{1/2} \qquad (1)$$

($W = E_0^2/8\pi$ is the pump-wave energy density, $N$ and $T_e$ are the concentration and temperature of the electrons, and $m$ and $M$ are the masses of the electrons and ions), when conditions for the development of modified decay instability are satisfied.[6] As shown by estimates, it is precisely this case which is realized in experiments[3] in which the effects of amplitude modulation and lengthening of the electromagnetic pulse become most clearly pronounced.

To illustrate the feasibility, in principle, of explaining the effects noted above, we investigate a very simple model problem concerning the development of modified decay instability in a homogeneous transparent plasma layer, using as an example the two-mode regime, when an electromagnetic wave incident in the direction of the $z$ axis and a plasma wave with maximum parametric-instability increment participate in the interaction.

The equations describing such an interaction are

$$\frac{\partial A_1}{\partial \tau} + v_1\frac{\partial A_1}{\partial \xi} + \gamma A_1 = -iA_2 s\, e^{i\Delta\tau},$$
$$\frac{\partial A_2}{\partial \tau} + v_2\frac{\partial A_2}{\partial \xi} + \gamma A_2 = -iA_1 s^* e^{-i\Delta\tau}, \qquad (2)$$
$$\frac{\partial^2 s}{\partial \tau^2} + \Gamma s = -A_1 A_2^* e^{-i\Delta\tau},$$

where $E_{1,2} = E_0 A_{1,2}\exp\{i\omega_{1,2}t - ik_{1,2}z\}$ are the electric fields of the high-frequency waves, $A_{1,2}(\xi,\tau)$ are the normalized slowly-varying components of the amplitude, $s = n\delta/N$, $n$ are the low-frequency perturbations of the plasma density, $\delta = \omega_{pe}^2/4\gamma_N(\omega_1,\omega_2)^{1/2}$, $\omega_{p\alpha} = 4\pi e^2 N/m_\alpha$, $v_{ph} = (k_1 - k_2)/\omega_1$, $\gamma_N = \omega_1(v^2/v_{ph}^2)(\omega_{pi}^2/\omega_1^2)^{1/3}$, $\kappa = k_1 - k_2$, $v_\sim = eE_0/m\omega_1$, $\tau = \gamma_N t$, $v_{1,2} = v_{gr}^{(1,2)}/c$, is the projection of the group velocities of the waves on the $z$ axis, $\xi = z\gamma_N/c$, $z$ is the coordinate transverse to the layer, $\gamma = v_{eff}/\gamma_N$ is the damping decrement, $\Delta = (\omega_1 - \omega_2)/\omega_1$, $\Gamma = 2\kappa^2 v_s^2/\gamma_N^2$, and $v_s = 2T_e/M$.

The system (2) was solved with a computer with different selected problem parameters and under the following initial and boundary conditions:

$$\tau = 0 \quad |A_1| = 1,\ |A_2| = A_{20} \ll 1,\ 0 \leqslant \xi \leqslant 1, \qquad s = \arg A_{1,2} = 0,$$
$$\xi = 0 \quad |A_1| = 1(v_1\tau) - 1[v_1(\tau - \tau_0)] \qquad 1(x) = \begin{cases} 1 & x \geqslant 0 \\ 0 & x \leqslant 0 \end{cases} \qquad (3)$$
$$s = \arg A_{1,2} = 0, \qquad |A_2| = A_{20} \ll A_1 \qquad v_2 = 0{,}01 v_1.$$

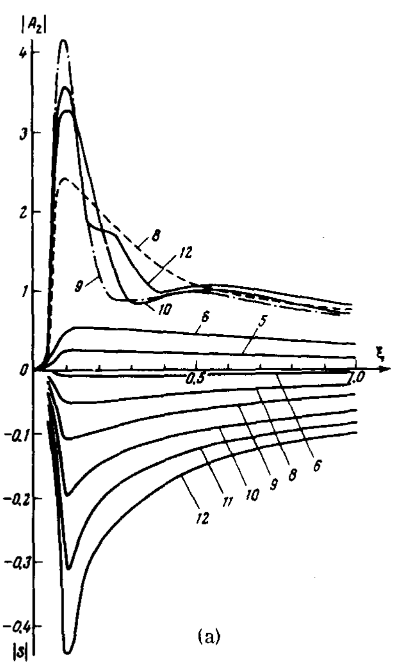


FIG. 1a. Distribution of the plasma-oscillation amplitude $|A_2|$ and of the density perturbations $|s|$ over the plasma layer at various instants of time.

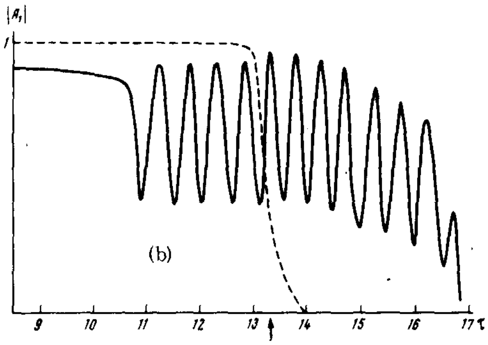


FIG. 1b. Effects of amplitude modulation and lengthening of the electromagnetic pulse (numerical calculation). The dashed curve shows the initial pulse.

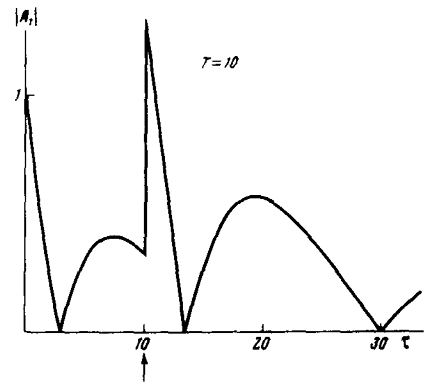


FIG. 2. The same as in Fig. 1b (analytic calculation). The arrow indicates the instant when the initial pulse is turned off.

Some examples of the numerical calculation at $\gamma = 0$ and $\Delta = 0$ are shown in Figs. 1a and 1b. The most typical feature is the strongly inhomogeneous distribution (over the layer) of the intensity of the plasma waves and of the low-frequency density perturbations (Fig. 1a). Effects of accumulation of the plasma turbulence in the layer are clearly pronounced ($|A_2|_{max} \approx 5$). Figure 1b shows a plot of the pump-wave amplitude against the time at the exit from the layer $\xi = 1$. What is typical here is the deep amplitude modulation due to the dynamic character of the process, and also the pulse-lengthening effects due to radiation of the plasma turbulence from the layer after the pump wave is turned off. A major role is played by the damping ($\gamma \neq 0$), which greatly increases the concentration of the plasma-wave energy and of the perturbed density on the leading edge of the layer, and also weakens the effects of energy accumulation and pulse lengthening after the pump is turned off.

Taking into account the qualitative similarity of the processes occurring in the two-mode and many-mode regimes,[7] we can propose that the picture described above remains in force, in its main features, in the more general case of multimode interaction, when the production of spatial harmonics of the plasma-wave field is taken into account.

To reveal the qualitative features of the considered interaction, we investigated the analytic solution of the first two equations of the system (2) for a specified periodic structure of the density perturbations $s = s_0$. This formulation of the problem is dictated by monotonic behavior and by the decrease of the relative growth rates of the perturbations $|s|$ in the numerical calculation (see Fig. 1a), and is also justified physically, inasmuch as in real experiments, under multiple or prolonged sounding, the density perturbations seem to accumulate in the interaction region up to a certain stationary level.

Let us write out the solution of Eqs. (2) for $A_1$ and $A_2$ at a given value $s = s_0$. In the simplest case $\gamma = v_2 = A_{20} = 0$ with the initial boundary conditions (3), the solution for $A_1$ is

$$A_1 = J_0\left[s_0 \gamma_N \sqrt{\frac{2z}{v_1}\left(t - \frac{z}{v_1}\right)}\right] - J_0\left[s_0 \gamma_N \sqrt{\frac{2z}{v_1}\left(t - T - \frac{z}{v_1}\right)}\right] 1\left(t - T - \frac{z}{v_1}\right) \qquad (4)$$

$$A_2 = \left(\frac{t - \frac{z}{v_1}}{z/v_1}\right)^{1/2} J_1\left[s_0 \gamma_N \sqrt{\frac{2z}{v_1}\left(t - \frac{z}{v_1}\right)}\right] - \left(\frac{t - T - \frac{z}{v_1}}{z/v_1}\right)^{1/2} J_1\left[s_0 \gamma_N \sqrt{\frac{2z}{v_1}\left(t - T - \frac{z}{v_1}\right)}\right] \times 1\left(t - T - \frac{z}{v_1}\right), \qquad (5)$$

where $J_{0,1}$ is a Bessel function of real argument, $T$ is the pump-pulse duration, and $1(x)$ is the Heaviside unit step function.

According to (4)–(5), at the exit from the layer $z = z_0$ there is deep amplitude modulation of the pump pulse with the characteristic period $T_M \approx v_1/2s_0^2\gamma_N^2 z_0 \approx 8v_1/\epsilon^2\omega_{pe}^2 z_0$, where $\epsilon = n_0/N$, $z_0$ is the layer thickness, and $n_0$ is the amplitude of the perturbations. Inside the layer, the plasma turbulence accumulates with time. After turning off the pump wave, a lengthening of the pulse by $\Delta T \sim T$ takes place (Fig. 2). Similar effects were observed in the experiments of Shlyuger *et al.*[3]

Let us make a few quantitative estimates based on the experimentally observed[3] modulation period $T_m \sim 2 \times 10^{-4}$ sec at a pulse duration $T \sim 6 \times 10^{-4}$ sec. Assuming $\omega_{pe}/2 \approx f_0 \sim 1.4 \times 10^6$ Hz, $T_m \approx 2 \times 10^{-4}$ sec and $v_1 \approx 10^{10}$ cm/sec, we obtain a value $n_0/N \approx 10^{-2}$ at $z_0 \approx 3$ km, which is perfectly acceptable for ionospheric conditions.


[1]W. F. Utlaut and R. Cohen, Science 174, 245 (1971).
[2]G. G. Getmantsev, G. P. Komrakov, Yu. S. Korobkov, L. F. Mironenko, N. A. Mityakov, V. O. Rapoport, V. Yu. Trakhtengerts, V. L. Frolov, and V. A. Cherepovitskiĭ, ZhETF Pis. Red. 18, 621 (1973) [JETP Lett. 18, 364 (1973)].
[3]I. S. Shlyuger, ZhETF Pis. Red. 19, 274 (1974) [JETP Lett. 19, 162 (1974)].
[4]V. V. Vas'kov and A. V. Gurevich, Zh. Eksp. Teor. Fiz. 64, 1272 (1973); 66, 1673 (1974) [Sov. Phys.-JETP 37, 646 (1973); 39, No. 5 (1974)].
[5]L. I. Al'ber, Z. N. Krotova, N. A. Mityakov, V. O. Rapoport,

and V. Yu. Trakhtengerts, Zh. Eksp. Fiz. **66**, 574 (1974) [Sov. Phys.-JETP **39**, No. 2 (1974)].
[6]A. G. Litvak, V. A. Petrukhina, and V. Yu. Trakhtengerts, ZhETF Pis. Red. **18**, 190 (1973) [JETP Lett. **18**, 111 (1973)].
[7]A. G. Litvak, V. Yu. Trakhtengerts, G. M. Fraiman, and T. N. Fedoseeva, ZhETF Pis. Red. **20**, 544 (1974) [JETP Lett. **20**, 248 (1974)].