Dragging effect in the transverse magnetoresistance of metals

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(Submitted July 15, 1974)
ZhETF Pis. Red. **20**, No. 9, 621–625 (November 5, 1974)

1. The possibility of observing an exponential temperature dependence of the resistance of metals at low temperatures, due to the dragging effect, encounters fundamental difficulties. Analysis has shown[1,2] that this possibility can be realized only in uncompensated metals with closed Fermi surfaces, and in the presence of defects, furthermore, only in those metals that have nearly-spherical Fermi surfaces (see also[3]). As a result, the phenomenon becomes restricted to the alkali metals.

However, the dragging effect should be clearly manifest in the magnetoresistance at low temperature. In the case of uncompensated metals with closed Fermi surfaces, this is due to the suppression,[4] in a strong magnetic field, of the electron distribution function responsible for the requirement that the Fermi surface be spherical.[2] In the case of compensated metals with closed Fermi surfaces, the magnetic field itself, in contrast to the electric field, produces a unified drift of electrons and holes, which in turn stimulates phonon drift. The stronger the phonon dragging, the less effective the scattering and the larger the transverse resistance. Thus, dragging can manifest itself in the magnetoresistance of all metals with closed Fermi surfaces.

It appears that the dragging effect in magnetoresistance has not been considered in the literature at all, with the exception of a recently published note[5] devoted to the analysis of the influence of the magnetic field on the diffusion of electrons to those regions of the Fermi surface from which the transfer is effectively produced.

2. The linearized system of kinetic equations for the electron-phonon system of a metal and a magnetic field takes the following form:

$$N^{\alpha} = \hat{P}^{e,ph}(\phi^{\alpha}, \chi^{\alpha}) + \hat{R}(\phi^{\alpha}) + \hat{M}(\phi^{\alpha}),$$
$$0 = \hat{P}^{ph,e}(\chi^{\alpha}, \phi^{\alpha}) + \hat{L}(\chi^{\alpha}). \tag{1}$$

Here $\hat{R}$ is the operator for the electron collision with defects, and $\hat{L}$ is the corresponding operator describing the phonon-phonon interaction and the interaction of the phonons with the defects. The remaining symbols are analogous to those used in[4] (α is the index of the Cartesian coordinate).

We confine ourselves to uniaxial or cubic crystals. We expand the correction to the electron (ϕ^{α}) and phonon (χ^{α}) distribution function in certain complete systems of functions $\{\psi_i\}$ and $\{\xi_{\nu}\}$, which depend on the generalized arguments $k \equiv k$, n (n is the number of the band) and $q \equiv q$, s (s is the number of the branch), respectively.

Solving the second equation of (1) with respect to the coefficients of the expansion of the function χ^{α} and substituting the obtained values in the first equation, we obtain for the expansion coefficients of the electron distribution function $a_j(\alpha)$:

$$\langle N^{\alpha}, \psi_i \rangle = \sum_j K_{ij} a_j(\alpha), \quad \hat{K} = \hat{\tilde{P}} + \hat{R} + \hat{M}, \tag{2}$$

where

$$\tilde{P}_{ij} = P_{ij} - \sum_{\nu\nu'} P'_{i\nu}(Q^{-1})_{\nu\nu'} P'_{\nu' j}. \tag{3}$$

Here P_{ij} and $P'_{\nu j}$ are the operator of the electron-phonon interaction at $\chi^{\alpha} = 0$ and $\phi^{\alpha} = 0$, respectively ($P'_{\nu j} = P'_{j\nu}$), and $\hat{Q}$ is the collision operator corresponding to the right-hand side of Eq. (1) at $\phi^{\alpha} = 0$.

3. We consider uncompensated metals. In this case Eq. (2) coincides exactly with the equation investigated in[4] provided the substitution $\hat{P} \to \hat{\tilde{P}}$ is made. This enables us to use directly the asymptotic behavior obtained in[4] for the resistance in a strong magnetic field.

We choose as the first three functions in the systems $\{\psi_i\}$ and $\{\xi_{\nu}\}$ the quasimomentum components k^{α} and q^{α} (if pieces of the Fermi surface emerge to the boundary of the Brillouin zone, it is necessary to use a periodic piecewise-linear function k^{α}, see[1]). Then, in accord with[4], if **H** is parallel to one of the principal axes (the 3 axis), we obtain for the transverse component of the resistance tensor along the principal axis 1

$$\rho^{11}_{H\to\infty} = \frac{1}{j^2}(\tilde{P}_{11} + P_{11}), \tag{4}$$

$$j = \langle N^{\alpha}, \psi_1 \rangle = e(N_e - N_h). \tag{5}$$

(The index i in the first three functions corresponds to the number of the axis).

If we neglect the scattering of the phonons by the defects, then we can easily show that $\tilde{P}_{11}$ vanishes in the absence of Umklapp processes in the electron-phonon system of the metal. As a result, as $\mathbf{H} \to \infty$, the temperature-dependent part of the resistance has at low temperatures an exponential dependence on T (for details see[2]) *regardless of the character of the electron scattering by the defects*. By the same token, it becomes possible, in principle, to observe manifestations of the dragging effect in uncompensated metals with anisotropic Fermi surfaces, manifestations that are masked in the measurement of the usual resistance at $\mathbf{H}=0$ by the power-law dependence of ρ on T, due to the anisotropic scattering of the electrons by the defects.[2,3]

4. In the case of compensated metals with closed Fermi surfaces, the quantity j in (5) vanishes identically. We therefore separate explicitly one more triad of functions ψ_i, obtained from one another by cyclic permutation of the coordinates $\psi_{\alpha'}$, which should correspond to the condition

$$< N^{\alpha}, \psi'_{\alpha_1} > = j'_{\alpha}\delta^{\alpha\alpha_1} \neq 0. \tag{6}$$

Then

$$<\psi'_{\alpha}|\hat{M}|\psi_{\alpha_1}> = -\frac{1}{c}e^{\alpha\alpha_1\gamma}j'_{\alpha}H^{\gamma}; \quad <\psi_{\alpha}|\hat{M}|\psi_{\alpha_1}> = -\frac{j}{c}e^{\alpha\alpha_1\gamma}H^{\gamma} = 0. \tag{7}$$

Following,[4] we stipulate that all the function ψ_i with the exception of the functions of the second triad be mutually orthogonal and orthogonal to N^{α}, and that they simultaneously satisfy the condition $\langle\psi'_{\alpha}|\hat{M}|\psi_j\rangle = 0$, $(j>6)$. The matrix $\hat{K}$ then has the following block structure

$$\hat{K} = \begin{pmatrix} \hat{F}^{11} & \hat{K}^{12} & \hat{F}^{(1)} \\ \hat{K}^{21} & \hat{K}^{22} & \hat{F}^{(2)} \\ \hat{F}^{(1)}_T & \hat{F}^{(2)}_T & \hat{K}^{(3)} \end{pmatrix}, \quad \hat{F} = \hat{\tilde{P}} + \hat{R}. \tag{8}$$

Here $\hat{K}^{ss'}$ and $\hat{F}^{ss'}$ are quadratic third-order matrices with elements equal to the matrix elements between the functions belonging to the first $(s=1)$ and second $(s=2)$ triads. The determination of the remaining blocks is automatic.

If we obtain a matrix $\hat{T}$ that is inverse to $\hat{K}$, then the resistance tensor is determined by the third-order matrix $(\hat{T}^{22})^{-1}$, where $\hat{T}^{22}$ is a block of the inverse matrix $\hat{T}$ and occupies the position of $\hat{K}^{22}$ (cf. [4]). As a result, we obtain in the general case

$$\rho^{\alpha\beta} = \frac{1}{j'_{\alpha}j'_{\beta}}[\hat{K}^{22} - \hat{K}^{21}(\hat{F}^{11} - \hat{F}^{(1)}(\hat{K}^{(3)})^{-1}\hat{F}^{(1)}_T)^{-1}\hat{K}^{21}$$
$$+\hat{F}^{(2)}(\hat{K}^{(3)} - \hat{F}^{(1)}_T(\hat{F}^{11})^{-1}\hat{F}^{(1)})^{-1}\hat{F}^{(1)}_T(\hat{K}^{(22)})^{-1}\hat{K}^{12}$$
$$+\hat{K}^{21}(\hat{K}^{22} - \hat{F}^{(1)}(\hat{K}^{(3)})^{-1}$$
$$\times\hat{F}^{(1)}_T)^{-1}\hat{F}^{(1)}(\hat{K}^{(3)})^{-1}\hat{F}^{(2)}_T - \hat{F}^{(2)}(\hat{K}^{(3)}$$
$$-\hat{F}^{(1)}_T(\hat{F}^{11})^{-1}\hat{F}^{(1)})^{-1}\hat{F}^{(2)}_T]_{\alpha\beta}. \tag{9}$$

The asymptotic behavior of the transverse resistance in a strong magnetic field is determined by the second term of (9), which gives a quadratic dependence of $\mathbf{H}$, in accord with the known general result.[6] If $\mathbf{H}$ is directed along a principal axis, then $(K^{(3)})^{-1}$ vanishes[4] and we arrive at the expression

$$\rho^{11}_{H\to\infty} = \frac{H^2}{c^2(\tilde{P}_{22} + R_{22})} \tag{10}$$

($\mathbf{H}$ is parallel to the 3 axis). Thus, the temperature-dependent part of the resistances (more accurately, of the coefficient of H^2) is again uniquely determined by the diagonal matrix element of the generalized collision operator for the pure drift piecewise-linear quasimomentum function ψ_2, although the dependence, naturally, is also inverse. It is interesting that the drift component that is singled out is precisely the one along the 2 axis, corresponding to a single drift of electrons and holes at $\mathbf{H}$ along the 3 axis and $\mathbf{E}$ along the 1 axis.

All the arguments advanced above concerning the exponential dependence on T at low temperatures and on the suppression of the role of the anisotropic scattering of the electrons by impurities under dragging conditions, connected with the peculiar symmetrization of the distribution function of the electrons in a strong magnetic field, actually make it possible to observe the dragging effects, which in principle is missing in the resistance of compensated metals at $\mathbf{H}=0$.

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Erratum: Manifestation of dragging effect in transverse magnetoresistance [JETP Lett. 20, No. 9, 284 (1974)]

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ZhETF Pis. Red. 20, No. 10, 692 (November 20, 1974)

It should be noted that for the case of compensated semimetals, the possible appearance of the dragging effect in the magnetoresistance was first analyzed in a paper by Gurevich and Korenblit [Fiz. Tverd. Tela **9**, 1195 (1967)] [Sov. Phys. -Solid State **9**, 932 (1967)].