New model of the anisotropic phase of superfluid He^3

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It is shown at a sufficiently large value of the exchange gain in He^3, there exists a possibility of pairing in states with spin $s=1$ with even orbital angular momenta. This does not contradict the Pauli principle, since the average many-time wave functions of the pairs (the anomalous Green's function) depends in odd manner on the difference between the temporal arguments.

The reasons why there are several superfluid phases in He^3, as well as the nature of the difference between the phases, are not yet clear. If it is assumed that pairing takes place in these phases with spin $s=1$, then it follows from microscopic calculations that while symmetry considerations admit in this case of the possibility of different phases, the minimum of the free energy should correspond under rather general assumptions to only the quasi-isotropic phase described by Balian and Werthammer.[1] Anderson and Brinkman[2] have assumed that this conclusion no longer holds if account is taken of the interaction via the spin-density fluctuations. The large value in He^3 is expected in connection with the fact that He^3 is close to a ferromagnetic transition, i.e., the exchange gain (the Stoner factor) is large:

$$S = \chi(0,0)/\chi_{\text{nonint}} = (1+F_0^{(a)})^{-1} >> 1 \tag{1}$$

(χ_{nonint} is the susceptibility of the noninteracting quasiparticles $\chi(0,\ 0)$ is the susceptibility at $k_0=0$ and $\mathbf{k}=0$, and $F_0^{(a)}$ is the Fermi-liquid amplitude of the exchange interaction at $l=0$). In an analysis of[2], the author has detected the possibility of a new type of anisotropic phase, which apparently has not been noted before. The present communication is devoted to a brief description of this possibility.

The pair condensate, as is well known, is described by an anomalous Green's function (a Gor'kov function). We use the temperature Green's functions in spinor-covariant form ($g_{\lambda\nu}=(i\sigma_y)_{\lambda\nu}$ is the metric tensor of spinor algebra, $\psi(1)=\psi(\tau_1\mathbf{r}_1)$, and τ is the temperature time)

$$F^{\alpha}_{\beta}(1,\ 2) = -\langle T_\tau(\psi^\alpha(1)\psi^\lambda(2)\rangle g_{\lambda\beta}. \tag{2}$$

For the triplet condensate, the Fourier transform of (2) and of the corresponding self-energy part (i.e., the gap matrix) is given by ($\omega_m=(2m+1)\pi T$):

$$F^{\alpha}_{\beta}(\omega_m,\ \mathbf{p}) = \mathbf{F}(\omega_m,\ \mathbf{p})(\vec{\sigma})^{\alpha}_{\beta};\quad \Delta^{\alpha}_{\beta}(\omega_m,\ \mathbf{p}) = \vec{\Delta}(\omega_m,\ \mathbf{p})(\vec{\sigma})^{\alpha}_{\beta}. \tag{3}$$

The commutation relations of the Fermi operators lead only to

$$\mathbf{F},\ \vec{\Delta}(2,\ 1) = -\mathbf{F},\ \vec{\Delta}(1,\ 2);\quad \mathbf{F},\ \vec{\Delta}(-\omega_m,\ -\mathbf{p}) = -\mathbf{F},\ \vec{\Delta}(\omega_m,\mathbf{p}). \tag{4}$$

It is customary to consider states for which the following relations hold:

$$\mathbf{F},\ \vec{\Delta}(\omega_m,\ \mathbf{p}) = -\mathbf{F},\ \vec{\Delta}(\omega_m,\ -\mathbf{p}) = \mathbf{F},\ \vec{\Delta}(-\omega_m,\ \mathbf{p}). \tag{4a}$$

We shall show, on the other hand, that there can exist states for which (4) is satisfied on account of an odd dependence of $\mathbf{F}$ or $\mathbf{\Delta}$ on $\tau_1-\tau_2$ or ω, and an *even* dependence relative to p:

$$\mathbf{F},\ \vec{\Delta}(\omega_m,\ \mathbf{p}) = \mathbf{F},\ \vec{\Delta}(\omega_m,\ -\mathbf{p}) = -\mathbf{F},\ \vec{\Delta}(-\omega_m,\ \mathbf{p}). \tag{4b}$$

For the states (4b) to be possible it is necessary that $\mathbf{F}$ and $\mathbf{\Delta}$ depend on ω. At $T_c \ll \epsilon_F$ (T_c is a transition temperature), $\mathbf{F}$ and $\mathbf{\Delta}$ are concentrated in the region $|\xi| \lesssim T_c \ll \epsilon_F$ (where $\xi=\epsilon_p-\mu$ and μ is the chemical potential). This means that there should exist a scale $\omega_S \ll \epsilon_F$ in order that a region exist at which there is still no dependence on ξ but there is a dependence on ω/ω_S. We shall show that this situation occurs under the condition (1) by virtue of the retarded character of the interaction via paramagon exchange; the scale is the characteristic frequency

$$\omega_S = \frac{2^{3/2}}{\pi}(p_F^2/m^*)S^{-3/2} \sim \epsilon_F S^{-3/2}. \tag{5}$$

(It is possible that in the states (4b) the pairs consist of two atoms with total spin $s=0$ and virtual paramagnons with total spins $s'=1$; then condition (4b) should be satisfied for the function (2) which contains integration with respect to the paramagnon coordinates).

We consider for the gap a linearized equation corresponding to the state of the condensation. At $T_c \ll \epsilon_F$ it is possible to exclude in the usual manner the regions $|\xi| > \xi_\lambda$ (where $T_c \ll \xi_\lambda \ll \epsilon_F$), and the equation takes the form

$$\vec{\Delta}(\omega, \mathbf{n}) = -T \sum_{(\omega')} \int_{-\xi_\lambda}^{\xi_\lambda} d\xi \frac{a^2}{\omega'^2 + \xi^2} \times \int (d\mathbf{n}')\rho_F V^{(t)}(\omega, \mathbf{n}; \omega', \mathbf{n}')\vec{\Delta}(\omega', \mathbf{n}'). \tag{6}$$

Here $V^{(t)}$ is the effective interaction in the triplet state, $\rho_F = m^* p_F/2\pi^2$, and $(d\mathbf{n}) = (1/4\pi)\sin\theta d\theta d\phi$. $V^{(t)}$ is expressed in terms of the vertex part in the crossing channel (particle—hole); using for this vertex part the equation known from the theory of Fermi liquids, and changing over to imaginary frequencies, we find that under condition (1) there occur in $V^{(t)}$ singularities at $|\mathbf{n} \pm \mathbf{n}'| \to 0$ and $|\omega \pm \omega'| \to 0$, in the form

$$a^2 \rho_F V^{(t)}\begin{pmatrix}\mathbf{n} \to \pm\mathbf{n}' \\ \omega \to \pm\omega'\end{pmatrix} \cong \frac{(\pm 1)}{4}\left(S^{-1}\frac{\pi m^*}{2p_F^2}\frac{|\omega \pm \omega'|}{|\mathbf{n} \pm \mathbf{n}'|} + b|\mathbf{n} \pm \mathbf{n}'|^2\right)^{-1}. \tag{7}$$

The constants a and b in (7) have the following meaning: a is the constant for the renormalization of the pole part of the Green's function; $p_F^{-2}b^{-1}$ is the square of the Ornstein-Zernike radius of the spin correlation ($b_{\text{nonint}} = 1/12$). The denominator in (7) is in essence an expansion of $[\chi(k_0, k) - \chi(0, 0)]/\chi_{\text{nonint}}$ at $|k_0/kv_F| \ll 1$, $(k/p_F)^2 \ll 1$, where $k_0 = i(\omega \pm \omega')$ and $k = p_F|\mathbf{n} \pm \mathbf{n}'|$. It follows from (7) that $V^{(t)}$ can be represented in the form

$$a^2 \rho_F V^{(t)} = W(\mathbf{n}\mathbf{n}') - \frac{1}{4}\left[\frac{1}{4b}\ln S + U\left(\frac{|\omega - \omega'|}{\omega_S}\right)\right]\delta(\mathbf{n} - \mathbf{n}') + \frac{1}{4}\left[\frac{1}{4b}\ln S + U\left(\frac{|\omega + \omega'|}{\omega_S}\right)\right]\delta(\mathbf{n} + \mathbf{n}'), \tag{8}$$

where $W(\mathbf{n}\cdot\mathbf{n}') = -W(-\mathbf{n}\cdot\mathbf{n}')$ is the regular part of $V^{(t)}$, and $U(|z|)$ is a function in the form

$$U(|z|) = \lim_{\lambda \to \infty}\left\{\int_0^\lambda \frac{u\,du}{1 + \frac{|z|}{u} + 2bu^2} - \frac{1}{2b}\ln\lambda\right\},$$

$$U(|z| \ll 1) \cong \frac{1}{2b}\ln 2b - \frac{1}{4}\pi|z|/\sqrt{2b}; \tag{9}$$

$$U(|z| \gg 1) \cong -\frac{1}{6b}\ln\frac{|z|}{2b}.$$

Equation (6) breaks up into two independent equations, namely $\mathbf{\Delta}(\omega, \mathbf{n}) = -\mathbf{\Delta}(\omega, -\mathbf{n}) = \mathbf{\Delta}(-\omega, \mathbf{n})$ in the subspace P' and $\mathbf{\Delta}(\omega, \mathbf{n}) = \mathbf{\Delta}(\omega, -\mathbf{n}) = -\mathbf{\Delta}(-\omega, \mathbf{n})$ in the subspace P''; these equations are invariant with respect to $V^{(t)}$ (8). On P'' we have $\delta(\mathbf{n}+\mathbf{n}')P'' = \delta(\mathbf{n}-\mathbf{n}')P''$ and $W(\mathbf{n}\cdot\mathbf{n}')P'' = 0$, and in (6) it is possible to integrate with respect to ξ in the range $\pm\infty$, since the cutoff scale is here $\omega_S \ll \epsilon_F$. Solutions of the type P'' give rise to states of the type (4b). We write out directly the nonlinearized equations for such states. The Green's functions in the states (4b) take the form

$$(G_\beta^\alpha(\omega, \mathbf{p}) = G(\omega, \mathbf{p})\delta_\beta^\alpha,\ \mathbf{n} = \mathbf{p}/|\mathbf{p}|,\ \xi = \epsilon_\mathbf{p} - \mu,\ \omega_m = (2m+1)\pi T,\ m = 0, \pm 1, \pm 2, \ldots),$$

$$G(\omega_m, \xi) = \frac{-(i\omega_m + \xi)}{\omega_m^2 + \xi^2 - |\Delta(\omega_m, \mathbf{n})|^2};\quad \mathrm{F}(\omega_m, \xi) = \frac{-\Delta(\omega_n, \mathbf{n})\vec{\nu}}{\omega_m^2 + \xi^2 - |\Delta(\omega_m, \mathbf{n})|^2}, \tag{10}$$

where ν is an arbitrary unit vector that singles out the direction in spin space. In the assumed approximation $\Delta(\omega_m, \mathbf{n})$ takes the form

$$\Delta(\omega_m, \mathbf{n}) = Te^{i\phi(\mathbf{n})}\Delta(\nu_m, T);\quad \phi(\mathbf{n}) = \phi(-\mathbf{n}) = \phi^*(\mathbf{n}), \tag{11}$$

where $\Delta(\nu_m) = -\Delta(-\nu_m)$ satisfies the equation $(\nu_m = (2m+1)\pi)$:

$$\Delta(\nu_m) = \frac{\pi}{4}\sum_{(m')}\left\{U\left(\frac{|\nu_m - \nu_{m'}|}{\omega_S/T}\right) - U\left(\frac{|\nu_m + \nu_{m'}|}{\omega_S/T}\right)\right\}\frac{\Delta(\nu_{m'})}{\sqrt{\nu_{m'}^2 - \Delta^2(\nu_{m'})}}. \tag{12}$$

Solutions of (12) with $\Delta \neq 0$ exist at $T/\omega_S \geq \lambda_c$, where λ_c is a certain critical value. The region of existence of the phase is given by

$$\lambda_-(S) \lesssim T/\epsilon_F \lesssim \lambda_+(S);\quad \lambda_-(S \gg 1) \sim S^{-3/2};\quad \lambda_+(S \gg 1) \cong \lambda_+(\infty). \tag{13}$$

Equation (12), which is accurate only at $T = O(S^{-1})$, yields $\lambda_+(\infty) = \infty$. To obtain $\lambda_+(\infty) < \infty$ it is necessary to make (7) more exact. In the general case $\lambda_+ = O(1)$, but there are models in which it is not necessary to go out of the region $T \ll \epsilon_F$, since $\lambda_+ \ll 1$ (not in terms of S^{-1} but in terms of other small parameters). When S decreases, the boundaries of the region (13) come closer together and this region should vanish at $S \leq S_c$. No account is taken in (12) of the fact that at $\Delta \neq 0$ the interaction $V^{(t)}$ itself depends on Δ (see[2]). Allowance for this fact does not change (13), but can lead to solutions with a spin symmetry different from (10). The degeneracy connected with the arbitrary form of the function $\phi(\mathbf{n})$ in (11) should be lifted also when account is taken of the corrections with respect to T/ϵ_F and of the Hamiltonian terms that introduce the spin-orbit coupling. Such a refinement is essential in order to find the possible types of symmetry of the phase and its physical (magnetic and superfluid) properties. These problems still remain to be investigated.

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[1]R. Balian and N. R. Werthammer, Phys. Rev. 131, 1553 (1963).
[2]P. W. Anderson and W. F. Brinkman, Phys. Rev. Lett. 30, 1108 (1973).