Interference filter for ultracold neutrons

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We describe an interference filter for neutrons with energy $\sim 10^{-8}$–10^{-4} eV. The filter consists of alternating layers of two different substances. The development and production of this filter can be of interest when it is necessary to filter out neutrons of required energy, and also to produce a neutron spectrometer with high resolving power in the indicated energy region.

In[1] we proposed an interference filter for neutrons of energy $\sim 10^{-8} - 10^{-4}$ eV. The filter constitutes a structure consisting of alternating layers (of thickness ~100 Å) of two different substances. The interaction of a neutron with such a structure can be described as an interaction of a neutron wave with potential in the form $U(x) = U_1$, at $x \in [n(a+b), n(a+b)+a]$; $U(x) = U_2$ at $x \in [n(a+b)+a,\ (n+1)(a+b)]$; and $U(x) \equiv 0$ at $x \geqslant N(a+b)+a$ and $x < 0$. $a \sim b \sim 100$ Å; n is the number of the period, $n = 0, 1, 2, \ldots,\ N \sim 10$.

An expression for U_1 (and an analogous expression for U_2) can be written in the form

$$U_1 = U_1' + iU_1'' = \frac{2\pi\hbar^2}{m} N_1 b_{\mathrm{coh}\,1} - i\frac{N_1\hbar}{2}\sigma_1 V.$$

Here $b_{\mathrm{coh}\,1}$ is the coherent neutron-scattering length, N_1 is the number of nuclei of the substance per unit volume, σ_1 is the cross section for absorption and inelastic scattering of the neutron, and V is the neutron velocity.

It is obvious that as $N \to \infty$ and at $U_1'' = U_2'' \equiv 0$ the interaction between the neutron wave and such a potential (in analogy with the interaction between an electron wave propagating in a crystal and a periodic electric potential) should lead to a band structure in the neutron energy spectrum.

In the case when N is finite one can expect the neutron spectrum to duplicate approximately the structure of an infinite periodic potential; the neutrons corresponding to "forbidden" bands should be reflected by the filter, and the remainder should pass through the filter.[1)] To find the concrete form of the spectrum in this case it is necessary to solve the Schrödinger equation with a potential of the type indicated above.

On each section $U(x) \equiv$ const, the solution of the equation can be expressed by a linear combination in the form

$$A_p \exp(ik_j x) + B_p \exp(-ik_j x) (\text{at } x < 0 \ \exp(ik_x x) + B\exp(-ik_x x)).$$

Here $k_j = (k_x^2 - 2mU_j/\hbar^2)^{1/2}$; k_x is the component of the wave vector of the neutron incident on the filter, m is the neutron mass, p is the number of the section, and $j = 1, 2$.

The coefficients A_p and B_p can be calculated by means of easily derived recurrence formulas with allowance for the requirement that the logarithmic derivative of the wave function of the neutron be continuous on the boundary between regions with different potentials.

Figure 1 shows the computer-calculated dependence of the reflection coefficient $|B|^2$ on V_x for the particular case $N = 20$, $a = b = 800$ Å, $U_1' = -0.50 \times 10^{-7}$ eV, $U_2' = 0.56 \times 10^{-7}$, $U_1'' = -0.23 \times 10^{-10}$ eV, and $U_2'' = -0.3 \times 10^{-12}$ eV. (The indicated values of the potentials correspond

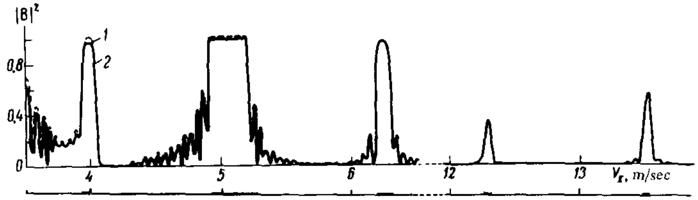

FIG. 1. Dependence of the neutron-wave reflection coefficient $|B|^2$ on the normal component of the neutron velocity V_x for the particular case $N = 20$, $a = b = 800$ Å, $U_1' = -0.5 \times 10^{-7}$ eV, $U_2' = 0.56 \times 10^{-7}$ eV. 1) $U_1'' = U_2'' \equiv 0$, 2) $U_1'' = -0.23 \times 10^{-10}$ eV, $U_2'' = -0.3 \times 10^{-12}$ eV. The additional axis under the velocity axis shows the "forbidden" band in the case of neutron motion in an infinite periodic potential with the same values of the parameters a, b, U_1', U_2', and $U_1'' = U_2'' \equiv 0$.

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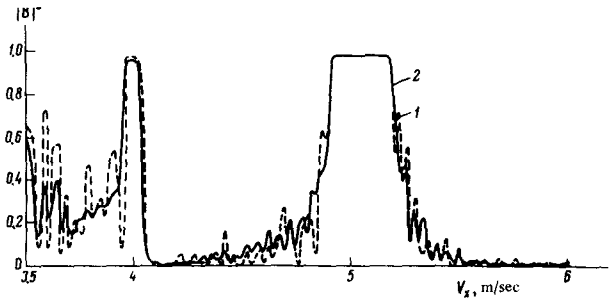

FIG. 2. Dependence of the neutron wave reflection coefficient $|B|^2$ on the normal component of the neutron velocity V_x: 1) arbitrary random set of layer thicknesses with mathematical expectation value 800 Å and variance 8 Å, 2) averaged over 10 random sets of thicknesses.

to a structure consisting of alternating layers of titanium and the tin isotope Sn^{118} of thickness 800 Å). It is seen from the figure that the behavior of $|B|^2$ at $N=20$ indeed duplicates the structure of the bands of the infinite periodic potential. The absorption and inelastic scattering leads to a negligible decrease of the amplitude $|B|^2$, mainly in the lowest bands.

If the filter is produced, for example, by thermal sputtering, then the layer thicknesses can be maintained only with a certain accuracy (~1%). We have therefore performed calculations that take into account random oscillations of the layer thicknesses. We assumed for these oscillations a normal distribution with a mathematical expectation value 800 Å and a variance 8 Å. The results of the calculations of $|B|^2$ for an arbitrary random set of layer thicknesses are shown in Fig. 2. It is seen from this figure that even for low-lying bands only the fine structure of the bands is altered. The same figure shows a plot of $|B|^2$ averaged over 10 random sets of thicknesses. This curve can be used to characterize the variance of $|B|^2$.

The development and production of structures of the described type can be of interest, for example, for filtering neutrons of the required energy, and also for the production of a neutron spectrometer with high resolution $\sim 10^{-8}-10^{-4}$ eV for low-energy neutrons.

The problem of spatial separation of a definite line in spectrometry can be solved with the aid of a magnetic prism[2] or by deflecting the neutron trajectory in a gravitational field. We note that it is possible to use both transmitted and reflected beams in instruments of this type. Particularly promising is the use of just a reflected neutron beam. To extract the reflected beam, the filter must be placed at a certain angle to the incident collimated neutron flux. Obviously, an interference filter makes it possible to obtain focused beams of reflected monoenergetic neutrons, provided the filter is constructed in the form of a cylindrical or spherical concave mirror.

In the case of a magnetic potential (if the adiabaticity conditions are satisfied) it is also possible to separate polarized-neutron lines.

It is easy to show[3] that the intensity of the reflected group of monoenergetic neutrons (in the case $mV_x^2/2 \gg U_2'$) is proportional to $V_x^{-1/2}$ and also to the reflection coefficient and to the intensity of the incident neutrons. For a Maxwellian distribution and $V_x \ll (kT/m)^{1/2}$, the latter quantity is proportional to V_x. Thus, the intensity of the reflected line is proportional to $V_x^{1/2}$.

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1) Preliminary results for the particular case $U_1=0$, $U_2 \equiv U_2'$ $=1.3\times 10^{-7}$ eV and $a=b=800$ Å were reported by us at the second International School on Neutron Physics in Alushta in April of this year. At the same paper, Schoenborn described a filter with a potential of sinusoidal form.

[1] A.V. Antonov, A.I. Isakov, M.V. Kazarnovskiĭ, V.I. Mikerov, and S.A. Startsev, Preprint FIAN No. 32, 1974.
[2] C.G. Shull, Trudy I Mezhdunar, shkoly po neĭtornnoĭ fizike (Proceedings for First International School on Neutron Physics), Alushta, 1969.
[3] L.D. Landau and E.M. Lifshitz, Krantovaya Mekhanika (Quantum Mechanics), Fizmatgiz, 1963 [Addison-Wesley, 1965].