Thermoelectric effect in anisotropic superconductors

V. L. Ginzburg and G. F. Zharkov

P. N. Lebedev Physics Institute, USSR Academy of Sciences
(Submitted October 1, 1974)
ZhETF Pis. Red. **20**, No. 10, 658–661 (November 20, 1974)

We discuss the thermoelectric effect that is produced in a homogeneous anisotropic superconductor in the presence of a temperature gradient.

The thermoelectric effect produced in an isotropic but inhomogeneous superconductor has been considered in[1,2] and in the other references cited therein. The thermoelectric effect (a circulating current and a corresponding magnetic field) should appear also in a homogeneous but anisotropic superconductor.[3] In connection with a recently undertaken attempt to observe this effect,[4] we have deemed it appropriate to turn to this case here, all the more since it is concluded in[4] that the theory of[3] does not agree with experiment.

Since the effect is maximal near the critical temperature T_c, there is every reason for confining oneself to the local approximation, in which the density of the superconducting current is given by[5,6]

$$j_k^{(s)} = -\frac{ie^*\hbar}{2m_k^*}\left(\psi^*\frac{\partial\psi}{\partial x_k} - \psi\frac{\partial\psi^*}{\partial x_k}\right) - \frac{(e^*)^2}{m_k^* c}A_k|\psi|^2,$$

where we use the system $k=1, 2, 3 = x', y', z'$ of the principal axes of the crystal, and there is of course no summation over k; we use the below the notation $\psi = \sqrt{N_s/2}\,e^{i\Phi}$, $e^* = 2e$, and $m_k^* = 2m\alpha_k$ (e and m are the charge and mass of the free electron, $N_s/2$ is the concentration of the superconducting pairs).

The density of the normal current is $j_k^{(n)} = b_{km}\partial T/\partial x_m$

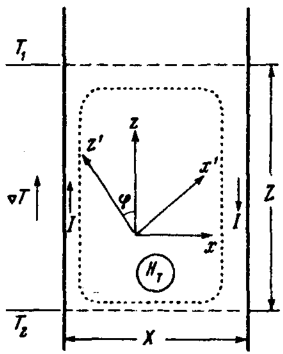

FIG. 1.

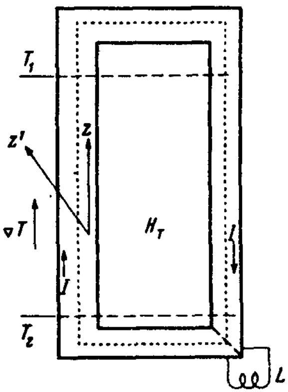

FIG. 2.

(see[3]), and consequently the density of the total current $\mathbf{j} = \mathbf{j}^{(s)} + \mathbf{j}^{(n)}$ is determined by the expression

$$\hat{\Lambda}\mathbf{j} = -\frac{1}{c}\mathbf{A} + \frac{\hbar}{2e}\nabla\phi + \hat{\Gamma}\nabla T. \qquad (1)$$

In the general case Λ_j is here a vector with components $\Lambda_{ik} j_k$, $\Lambda_{ik} = \Lambda\alpha_{ik}$, $\Lambda = m/e^2N_s = 4\pi\delta^2/c^2$, $\alpha_{ik} = m_{ik}/m$ ($\alpha_{ik} = \alpha_k\delta_{ik}$ in the case of the principal axes), and $\Gamma_{ik} = \Lambda_{im} b_{mk}$.

We consider now a sample in the form of a plate, under conditions that are clear from Fig. 1 (x' and z' are the symmetry axes of the crystal, and the z axis is directed along ∇T). We are interested in the temperature region near T_c, where the effect is maximal. Then $N_s \propto T_c - T$ and $\delta^2 = \delta_0^2(1 - T/T_c)^{-1}$. We neglect the weak dependence of a_{ik} and b_{ik} on the temperature. In addition, we assume that ∇T is constant.[1)] From (1) and from the equations $\mathrm{curl}\mathbf{A} = \mathbf{H}$ and $\mathrm{curl}\mathbf{H} = 4\pi\mathbf{j}/c$ we obtain

$$\mathbf{H} = \mathbf{H}_T - c\,\mathrm{rot}\,\hat{\Lambda}\mathbf{j}, \qquad \mathbf{H}_T \equiv c\,\mathrm{rot}\,\hat{\Gamma}\nabla T, \qquad (2)$$

$$\mathrm{rot}\,\mathrm{rot}\,\hat{\Lambda}\mathbf{j} + 4\pi c^{-2}\mathbf{j} = \mathbf{F}, \qquad \mathbf{F} \equiv \mathrm{rot}\,\mathrm{rot}\,\hat{\Gamma}\nabla T. \qquad (3)$$

When the foregoing is taken into account, $\mathbf{H}_T$ varies slowly with z, owing to the dependence of T on z. We assume that $\mathbf{H}_T$ is constant and $\mathbf{F} = 0$. It is then easy to verify that the solution of the system (2), (3) takes the form $\mathbf{H} = \mathbf{H}_T + \mathbf{H}'$ and $\mathbf{j} \sim \mathbf{j}_0 e^{-x/\delta}$, where $\mathbf{H}'$ and $\mathbf{j}$ decrease exponentially in the interior of the sample (cf. [3]). On the boundary with the vacuum we have $\mathbf{H} = 0$ and the current is $j = j_z \sim (c^2/4\pi\delta)(d\Gamma_{xz}/dz)(dT/dz)$.

Let now the term $\mathbf{F}$ in (3) differ from zero. The associated increment of the current is $j \sim (c^2/4\pi)(d^2\Gamma_{xz}/dz^2)(dT/dz)$. Therefore the role of the term $\mathbf{F}$ in (3) turns out to be small under the condition $\delta_0 T_c^{-1}(1 - T/T_c)^{-3/2} dT/dz \ll 1$, which is satisfied even if $dT/dz \sim 0.1$ and $1 - T/T_c \sim 10^{-4}$ ($\delta_0 \approx 2.5\times10^{-6}$ cm for tin). Consequently, under more realistic conditions, for example at $1 - T/T_c \sim 10^{-2}$, it can be readily assumed that $\mathbf{j} = 0$ in the interior of the superconductor (at a depth $\Delta x \gg \delta$). The field in the superconductor, according to (2), is then given by

$$H = H_T = c\frac{d\Gamma_{xz}}{dz}\frac{dT}{dz} = \frac{4\pi}{c}\delta_0^2\frac{a_{xx}b_{xz} + a_{xz}b_{zz}}{T_c(1 - T/T_c)^2}\left(\frac{dT}{dz}\right)^2. \qquad (4)$$

At $\delta_0 = 2.5\times10^{-6}$ cm, $T_c = 3.72$ °K (tin), $a_{xx}b_{xz} + a_{xz}b_{zz} = (b_{z'}a_{z'} - b_{x'}a_{x'})\sin\theta\cos\theta \lesssim b \sim 10^{11}$ cgs, we obtain $H \lesssim 10^{-10}(1 - T/T_c)^{-2}(dT/dz)^2$ so that $H \leqslant 10^{-8}$ Oe even at $1 - T/T_c' \sim 10^{-2}$ and $dT/dz \sim 0.1$.

Formula (4) agrees, apart from the notation, with formula (19) of[3], but was derived more simply and under more general assumptions. The estimate given in[3] also points to small values of H. According to[7], however, the estimate given in[4] for the field H is in error, since the coefficient b_{ik} in[7] was overestimated by several orders of magnitude (see[8]). It is obvious in general that Eq. (4) contains independently observable quantities and that the most reliable method of estimating H should be based on the use of the measured values of δ, T_c, b_k, and a_k, and not on calculations.

Consequently it is clear that the measurements of[4] do not contradict the theory, since the observed field is much stronger than the field (4) and is apparently due to other factors. In addition to the possible field sources discussed in[4], we call attention to the role of the inhomogeneity of the crystal (for example as a result of deformations). As is already clear from (2), in an inhomogeneous sample, when Γ_{ik} depend on the coordinates, a field proportional to ∇T is produced,[2)] as was indeed observed in[4].

Since the field H_T is quite weak, its direct measurement is difficult. The total current flowing through the sample is, however, $I = j_0\delta Y = (c/4\pi)H_T Y$ (X, Y, Z are the corresponding dimensions of the crystal, and it is assumed that Y is large enough), and the current I in a sufficiently large sample can be large. In this connection it is convenient to measure not the field H_T, but the current I, using a doubly-connected crystal for this purpose (Fig. 2). The, integrating (1) over the contour with $\mathbf{j} = 0$ (dashed), we obtain (cf. [1]).

$$\Phi = \oint \mathbf{A}d\mathbf{s} = \Phi_T + \Phi^{(0)}, \quad \Phi_T = H_T XZ,$$
$$\Phi^{(0)} = \frac{\pi\hbar c}{e}n, \quad n = 0, 1, 2 \ldots \qquad (5)$$

At $n = 0$ the result (5) coincides with (4). By cutting the crystal and passing a current I through an auxiliary superconducting coil L (Fig. 2), we can observe directly the field of the current I. This method is convenient also because the field in the coil L can be made much stronger than the field H_T by simply increasing the number of its turns.[3)] Additional enhancement of the effect can be obtained by inserting a superconducting core into an opening in the crystal (see Fig. 2). Finally, the total flux Φ can in principle turn out to be sufficient for the measurement of the emf when the circuit is opened. The discussed thermoelectric effect due to the anisotropy can be separated from the effect due to the inhomogeneities and to a number of other factors, by recognizing that the flux Φ is proportional to $XZ(dT/dz)^2$, whereas the other effects are proportional to dT/dz and generally speaking should not increase with increasing X. It is easily seen that the flux (5) can be of the same order of or even larger than the flux produced in an inhomogeneous superconducting circuit.[1,8]. The latter flux was already observed in experiment.[9]

Since we have discussed in[1,2] also the thermomechanical circulation effect in a super fluid liquid, we note that in superfluid He^3 (and in a neutron liquid in the case of production of pairs with nonzero angular momentum) it is necessary to take the anisotropy of the liquid into account (in the presence of boundaries or of a magnetic field). This can lead to the appearance of new circulation effects, due to the inhomogeneity of the temperature. Analogous circulation effects, but having no quantum features, can probably appear also in ordinary liquid crystals.

[1)]In the absence of heat sources in the sample itself we have $\mathrm{div}\hat{\kappa}\nabla T = 0$. Since the thermal-conductivity coefficients $\kappa_{ik}(T)$ remain finite as $T \to T_c$, it is probably possible in a number of cases to neglect, with good approximation, the temperature dependence and the anisotropy of the thermal conductivity.

[2)]By separating the temperature factor $f(z) = (1 - T/T_c)^{-1}$ and writing down $c\hat{\Gamma}\nabla T = \mathbf{a}f(z)$, we have $\mathbf{H}_T = \mathrm{curl}\ \mathbf{a}f = f\ \mathrm{curl}\ \mathbf{a} + \nabla f \times \mathbf{a}$. In the case of a homogeneous anisotropic superconductor, the vector $\mathbf{a}$ is constant and $\mathbf{H}_T = \nabla f \times \mathbf{a}$. In the inhomogeneous but isotropic case we have $\mathbf{a} \parallel \nabla f$ and $\mathbf{H}_T = f\ \mathrm{curl}\ \mathbf{a}$. In the general case both these terms differ from zero. The first gives the contribution $\sim(\nabla T)^2$ and the second a contribution $\sim\nabla T$.

[3)]This remark is valid also for an experiment with an inhomogeneous sample.[9] The flux Φ_L in the coil and the flux $\sim \Phi_T$ inside the superconducting circuit can differ in magnitude and in direction.

[1]V. L. Ginzburg, G. F. Zharkov, and A. A. Sobyanin, ZhETF Pis. Red. 20, 223 (1974) [JETP Lett. 20, 97 (1974)].
[2]G. F. Zharkov and A. A. Sobyanin, ZhETF Pis. Red. 20, 163 (1974) [JETP Lett. 20, 69 (1974)].
[3]V. L. Ginzburg, Zh. Eksp. Teor. Fiz. 14, 177 (1944); Journ. Phys. USSR 8, 148 (1944).
[4]P. M. Selzer, Dissertation, Stanford University, 1974; P. M. Selzer and W. M. Fairbank, Phys. Lett, A48, 279 (1974).
[5]V. L. Ginzburg, Zh. Eksp. Teor. Fiz. 23, 236 (1952).
[6]L. P. Gor'kov and T. I. Melik-Barkhudarov, Zh. Eksp. Teor. Fiz. 45, 1493 (1962) [Sov. Phys.-JETP 18, 1031 (1963)].
[7]V. Z. Kresin and V. A. Litovchenko, Zh. Eksp. Teor. Fiz. 53, 2154 (1967) [Sov. Phys.-JETP 26, 1216 (1968)].
[8]Yu. M. Gal'perin, V. L. Gurevich, and V. I. Kozub, Zh. Eksp. Teor. Fiz. 66, 1387 (1974) [Sov. Phys.-JETP 39, N0. 4 (1974)].
[9]N. V. Zavaritskiĭ, ZhETF Pis. Red. 19, 205 (1974) [JETP Lett. 19, 126 (1974)].