Possible experimental verification of microcausality in electroproduction

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(Submitted September 19, 1974; resubmitted October 7, 1974)
ZhETF Pis. Red. 20, No. 10, 671–674 (November 20, 1974)

It is shown on the basis of the results obtained in [4] that the Mellin transform of the forward scattering amplitude decreases exponentially. This statement admits, in principle, of experimental verification.

The microcausality principle has been verified so far only in forward elastic scattering of pions by nucleons. For this case there exist rigorously proved dispersion relations, [1–3] and these were verified experimentally.

The purpose of the present paper is to indicate one more possibility, in principle, of experimentally verifying satisfaction of microcausality, but this time in the electroproduction process.

In[4] we studied the requirements imposed by microcausality on the electroproduction process. It was demonstrated that the microcausality requirement is equivalent to the requirement that the amplitudes of the virtual Compton effect be analytic in the square of the photon mass. [5,6] More accurately, in order for microcausality to be satisfied in the electroproduction process it is necessary and sufficient that the functions

$$F_1\left(\frac{\nu^2}{q^2},\, q'^2\right) = 2\int_{-q'^2/2}^{\infty} \frac{\nu' q'^2 - 2\left(\frac{\nu^2 q'^2}{q^2} - \nu'^2\right)\left(\sqrt{\nu'^2 - q'^2} - \nu'\right)}{\nu'^2 - \nu^2(q'^2/q^2)}\, W_1(\nu', q'^2)\, d\nu' \tag{1}$$

and

$$F_2\left(\frac{\nu^2}{q^2},\, q'^2\right) = 2\int_{-q'^2/2}^{\infty} \frac{\nu' q'^2 - 2\left(\frac{\nu^2 q'^2}{q^2} - \nu'^2\right)\left(\sqrt{\nu'^2 - q'^2} - \nu'\right)}{\nu'^2 - \nu^2(q'^2/q^2)} \times \frac{\nu'^2 - q'^2}{q'^2}\, W_2(\nu', q'^2)\, d\nu' \tag{2}$$

$$m_p = 1$$

with real positive ν^2/q^2 be analytic functions in the variable q'^2 with a cut $[0, \infty]$.

The functions $F_1(\nu^2/q^2, q'^2)$ and $F_2(\nu^2/q^2, q'^2)$ are connected with the forward scattering amplitudes of a vertual γ quantum of mass q^2 by the formulas

$$T_1(\nu, q^2) = \frac{1}{q^2} F_1\left(\frac{\nu^2}{q^2},\, q^2 + oi\right), \tag{3}$$

$$T_2(\nu, q^2) = \frac{1}{\nu^2 - q^2} F_2\left(\frac{\nu^2}{q^2},\, q^2 + oi\right). \tag{4}$$

The gist of the proposed experimental verification that the microcausality requirements are satisfied in the electroproduction process is based on the theorem that the Mellin transform of an analytic function with a cut $[0, \infty|$ decreases exponentially with an exponent equal to π. Application of this theorem to the electroproduction process yields

$$\left|\int_0^{\infty} F_i\left(\frac{\nu^2}{q^2},\, q'^2\right)(-q'^2)^{i\alpha - 1}\, d(-q'^2)\right| \to C_i\left(\alpha,\, \frac{\nu^2}{q^2}\right) e^{-\pi\alpha} \tag{5}$$

$i = 1$ or 2, and $C_i(\alpha, \nu^2/q^2)$ are unknown functions of α and ν^2/q^2.

If $\alpha \gg 1$, then $C_i(\alpha, \nu^2/q^2)$ are slowly varying functions of α. It can be shown, on the basis of formula (3) of[4], that at $\alpha \gg 1$ the values of $C_i(\alpha, \nu^2/q^2)$ vary with increasing α not faster than in power-law fashion.

With respect to the comparison of formula (5) with experiment, we note the following: the functions $F_i(\nu^2/q^2, q'^2)$, defined by formulas (1) and (2), are expressed in terms of integrals of inelastic form factors $W_i(\nu, q^2)$, and can therefore be measured. Consequently, the integral in (5) can be measured. The accuracy with which the functions $W_i(\nu, q^2)$ can be measured cannot exceed 1%, owing to the presence of two-phonon exchange. Therefore the integral in (5) can be measured with accuracy up to 1%.

Even at $\alpha = 1.5$ we have $e^{-\pi\alpha} < 10^{-2}$. If the factors $C_i(\alpha, \nu^2/q^2)$ are not anomalously large, then the integral in (5) can be equated to zero already at $\alpha = 1.5$, and the resultant formulas can be regarded as sum rules of sorts. If $C_i(\alpha, \nu^2/q^2)$ turn out to be anomalously large, then an analogous sum rule will be obtained at large α. In this case one can hope to find also an exponential dependence of the integral (5) on α. The latter case can occur only if the functions $F_i(\nu^2/q^2, q'^2)$ are oscillating functions of q'^2.

Another possible verification of the microcausality requirements in the electroproduction process is to check on the satisfaction of the inequality (12) of[6].

The following circumstance must be emphasized: the inelastic form factors $W_i(\nu', q'^2)$ are not analytic functions. Only the integrals (1) and (2) of these functions are analytic. The fact that these integrals are analytic functions of q'^2 is equivalent to the requirement that microcausality be satisfied in electroproduction, so that a check on their analyticity in q'^2 is extremely important.

In conclusion, I am deeply grateful to B. L. Ioffe, A. I. Komech, and I. S. Shapiro for interest in the work and for useful remarks.

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