Isothermal domains in quasi-one-dimensional superconductors

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It is shown that the usual generalization of the Ginzburg–Landau equations to conclude the nonstationary case leads directly to the possible existence of domain boundaries of the superconducting and normal phases in a homogeneous quasi-one-dimensional superconductor. The current that must flow through the conductor for such a boundary to be in equilibrium is somewhat smaller than the critical pair-breaking current. Thus, equilibrium between the current-induced domains in the superconductor can exist also without the thermal effect discussed by Volkov and Kogan.

1. As shown by experiments[1] performed with long and narrow (width $\approx\xi$) strips of thin (thick $\ll\xi$) superconducting films, the restoration of the superconducting state in such quasi-one-dimensional samples with increasing current through the samples does not proceed in the manner predicted in the theoretical papers.[2] Namely, the appearance of a superconducting (S) phase occurs not at very small currents J through the conductor, but at values of J less than but comparable with the critical pair-breaking current J_c. The voltage on the sample decreases from the normal value JR_N to zero at a practically constant value of the current $J = J_0 < J_c$. It is natural to attribute such a picture to the propagation of S-phase domains from the superconducting electrodes towards the core of the sample, if it is known that stationary S-N walls of such domains can exist at $J = J_0$.

It was shown in[3] that the stability of such a boundary at $J < J_c$ can be ensured by the propagation of heat generated by the current in the N phase. However, according to the statement of the authors of[1], the sample heating in their experiments was quite small.

The purpose of the present article is to report that the existence of a stationary S-N boundary in a quasi-one-dimensional superconductor at a current $J = J_0 < J_c$ follows directly from the usual[2,4] equations for the isothermal case.

2. The usual one-dimensional nonstationary Ginzburg-Landau equations[2] for the order parameter $\psi = \Delta/\Delta_0$ and for the total electric-field potential μ can be expressed in the form

$$u\left(\frac{\partial}{\partial t} + i\mu\right)\psi = \left(\frac{\partial^2}{\partial x^2} + 1 - |\psi|^2\right)\psi ,$$
$$J = \mathrm{Im}\left(\psi^* \frac{\partial\psi}{\partial x}\right) - \frac{\partial\mu}{\partial x} , \quad (1)$$

where x is the coordinate along the conductor, normalized to $\xi(T)$, and u is the ratio of the order-parameter relaxation time t_ψ to the current relaxation time t_j. For the "dirty limit" we have $u = 12$ in the case of paramagnetic impurities and $u = \pi^4/14\zeta(3) \approx 5.79$ for nonmagnetic impurities. The time is normalized to t_j. The critical current J_0 in our normalization is equal to $\sqrt{4/27} \approx 0.385$. The functions $\psi(x)$ and $\mu(x)$, which describe a boundary located far enough away ($\gg\xi$) from neighboring boundaries, should satisfy the system (1) with the boundary conditions

$$|\psi(-\infty)| = \psi_1, \quad \psi(+\infty) = 0,$$
$$\mu(-\infty) = 0, \quad \frac{\partial\mu}{\partial x}(+\infty) = -J, \quad (2)$$

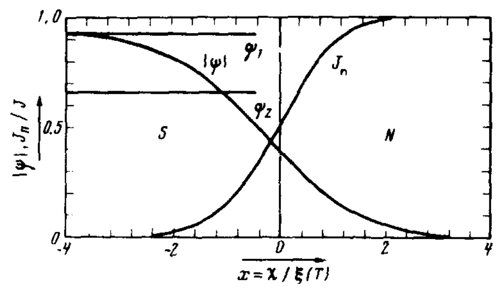
where ψ_1 is the larger positive root of the equation

$$\psi^2(1-\psi^2)^{1/2} = J. \quad (3)$$

The value of J_0 should be determined in this case from the conditions of zero velocity of the boundary: $(\partial/\partial t) = 0$.[4] The position of the boundary on a filament is of course indeterminate in this case, so that it can be fixed by imposing an additional condition, say $\partial\mu/\partial x(0) = -J/2$.

3. Since it turns out that the length of the stationary S-N boundary is of the order of unity (ξ), all the terms are significant in the right-hand sides of (1), and an analytic expression can be obtained only for the asymptotic values as $x \to \pm\infty$. Therefore Eqs. (1) with boundary conditions (2) were solved with a computer by the usual difference method for concrete values of the parameters u and J. Among the advantages of computer solution in this case we can include the fact that it is possible not only to find the shape of the stationary boundary $\psi(x)$, $\mu(x)$, but to verify directly the stability of this solution, since the computer in fact simulates the process (1).

The figure shows the domain-boundary shape calculated in this manner for $u = 5.79$; it turns out that J_0



Plots of ψ and of the normal current component ($J_N = -\partial\mu/\partial x$ against the coordinate x along the superconductor at the value of the current ($J = J_0$) at which the S-N boundary is immobile. $u = 5.79$, $J_0 \approx 0.335$, $\psi_1 \approx 0.92$, $\psi_2 \approx 0.67$.

≈ 0.335. The influence of the change of the coefficient u on the value of J_0 is still to be determined, but it is easy to show that when u tends to infinity, J_0 tends to zero like $u^{1/3}$.

4. From the data given in the second reference of[1] (Fig. 7) for a tin film it follows that at temperatures close to critical the values of J_0/J_c range from 0.72 at $T/T_c = 0.95$ to 0.55 at $T/T_c = 0.9$ (0.87 according to the theory). It can be assumed that the deviation of J_0/J_c from the calculated value is due either to the inevitable influence of the heat during the decrease of T[2] or to the influence of anomalous terms which are not taken into account in (1) and lead to an effective increase of t_ψ and u.[5]

5. It is easy to understand the physical cause of the existence of a stationary S-N boundary. As is well known, for currents smaller than $J_c(\psi_1 > \sqrt{2/3})$, a homogeneous ($\partial/\partial x = 0$) superconducting state appears if the initial value of $|\psi|$ is smaller than ψ_2, where ψ_2 is the smaller positive root of (3), and settles at the equilibrium level ψ_1 in the opposite case. Therefore the boundaries are at equilibrium when the suppression of the order parameter by the current at small values of $|\psi|$ in the N domain is balanced by the tendency of $|\psi|$ to the equilibrium level in the S domain.

6. If the current is not equal to J_0, the boundary propagates with velocity v and becomes slightly deformed as it moves. If v is measured in the direction of the S phase, then v is a monotonically increasing function of the difference $J - J_0$. It is easily understood that $v \to -\infty$ as $J \to 0$ and $v \to +\infty$ as $J \to J_c$. At currents near J_0 one can introduce a "viscosity coefficient"

$$\eta = \left(\frac{dv}{dJ}\right)^{-1}_{J=J_0}, \qquad (4)$$

which turns out to be close to 0.7 at $\mu = 5.79$.

7. As already mentioned, the results explain naturally the form of the current-voltage characteristics of the samples investigated in[7].

Moreover, we consider it possible to explain, in terms of the S and N domains, also the steplike structure of the current-voltage characteristics of other quasi-one-dimensional objects, such as whisker crystals.[6] To be sure, within the framework of Eqs. (1) it is impossible to obtain periodic solutions capable of explaining the domain structure of such objects.[1)] It turns out, however, that allowance for the two-dimensional character of the sample, even in first-order approximation, leads to the appearance of repulsion between neighboring N domains, which stabilizes the periodic domain structure and goes over, with further increase of the sample width, to the usual repulsion between two neighboring rows of vortices.

Thus, there is hope for explaining many observed effects in quasi-one-dimensional superconductors in terms of the motion and interaction of current-induced domains of the normal and superconducting phases.

[1)]The solutions proposed in[7] do not in fact satisfy these equations.

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