Nonlinear interaction of diffracted light beams in a medium with quadratic nonlinearity: mutual focusing of beams and limitation on the efficiency of optical frequency converters

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We report new nonlinear phenomena revealed by numerical experiments on the resonant interaction of two waves. The results explain many phenomena observed in experiments on the frequency doubling of high-power radiation.

1. We report in this article new nonlinear wave phenomena that are produced when two diffracted waves interact resonantly. Numerical experiments on second-harmonic generation of light beams have revealed for the first time the phenomena of mutual focusing and coupled waveguide propagation of beams in a medium

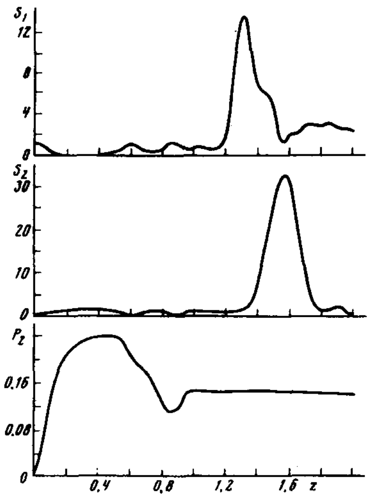

FIG. 1. Mutual focusing of beams behind a nonlinear layer. The plots represent the intensities $S_{1,2}$ of the fundamental radiation and of the second harmonic on the z axis, and the power $P_2(z)$: $D_1 = 0.02$, $\gamma = 15$, $\Delta = 0$, $A_1(r, 0) = \exp(-r^2)$.

with quadratic nonlinearity, in contrast to the known phenomenon of self-focusing and waveguide propagation in media with cubic nonlinearity. The efficiency of high-power frequency doublers is limited by the energy transfer that develops as a consequence of diffraction-induced randomization of the phases of the beams.

The results of numerical experiments explain many phenomena observed in experiments on frequency doubling of high-power laser radiation, such as the formation of a ring structure of Gaussian beams[2] and breakdown of crystals following harmonic generation,[3] as being due to development of mutual focusing in the crystal; the low efficiencies[2] are attributed to backward transfer of energy near the beam axis.

2. The interaction of diffracting beams of the first and second harmonic will be described in the quasioptical approximation by the system of equations[1,4]

$$\frac{\partial A_1}{\partial z} + i D_1 \Delta_\perp A_1 = -i\gamma A_1^* A_2 e^{-i\Delta z}, \tag{1}$$

$$\frac{\partial A_2}{\partial z} + i D_2 \Delta_\perp A_2 = -i\gamma A_1^2 e^{i\Delta z}, \tag{2}$$

where A_j are the normalized amplitudes of the beams, the intensities being $S_j = |A_j|^2$; z is measured along the length L of the crystal, $0 < z < 1$, $\Delta = (k_2 - 2k_1)L$ is the normalized mismatch of the phase velocities, $D_j = L/2k_j a^2$, a is the transverse dimension of the beam, γ is the coefficient of nonlinear coupling, and $\Delta_\perp$ is the Laplace operator in the plane perpendicular to the z axis.

The system (1)—(2) has integrals of motion $I_1 = P_1 + P_2$, $P_j = \int\int S_j\, dx\, dy$ is the conservation law of the total beam energy, and a phase asynchronism integral $I_2 = \int\int [2D_1|\nabla_\perp A_1|^2 + D_2|\nabla_\perp A_2|^2 - \nabla^2|A_2|^2 - \gamma(A_1^2 A_2^* + A_1^{*2} A_2)]\, dx\, dy$.

A numerical solution of the system (1)—(2) was obtained for cylindrically symmetrical beams with initial Gaussian distribution

$$A_1(r, 0) = a \exp\left\{-\frac{r^2}{a^2}(1 - i\beta)\right\}, \quad A_2(r, 0) = 0 \tag{3}$$

within a cylinder $r < R$ at $A_1(R, z) = A_2(R, z) = 0$. Conservative difference schemes were used for the calculation.[7] The value of R was chosen to be large enough ($R \gg a$) to be able to disregard the influence of the side boundary. The calculations were carried out up to $z = 2$, in order to be able to trace the behavior of the waves after emergence from the crystal at $z > 1$.

3. Allowance for the diffraction in the standard frequency-doubling problem leads to qualitatively new effects. As is well known, it follows from the theory of plane waves that 100% of the fundamental radiation energy can be converted into the second harmonic in the case of exact phase matching ($\Delta = 0$). The numerical experiments performed with $\Delta = 0$ have shown, however, that at a sufficiently short nonlinear interaction length ($L_{nl} \sim \gamma^{-1}$), the energy exchange between bounded beams acquires an entirely different character. The results of the numerical calculations are shown in Figs. 1 and 2. We see that energy is returned by the second harmonic to the fundamental wave. This transfer first starts on the beam axis, and then spreads over an ever-increasing region; the efficiency of the doubler is then decreased. The reasons for the backward energy transfer is the diffraction collapse of the phases of the beams. For the characteristic energy-transfer lengths we can propose the empirical formulas

$$z_S = \gamma^{-1} \ln \frac{\gamma}{2D_1}, \quad z_P = \gamma^{-1} \ln \frac{2\gamma}{D_1}, \tag{4}$$

which should be compared with the formula for the transfer length in the case of plane waves with phase difference $\Delta \neq 0$[5,6]

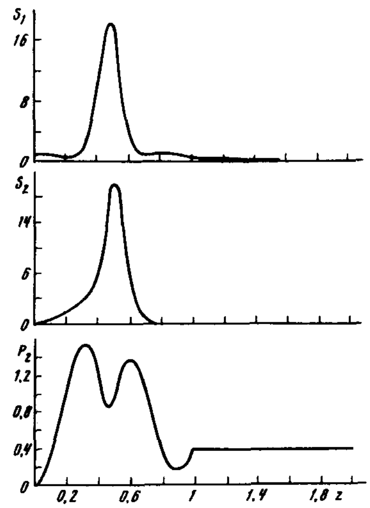

FIG. 2. Mutual focusing of waves in the case of a beam of fundamental radiation focused at $z = 0.5$. $S_{1,2}$ are the intensities on the z axis, $P_2(z)$ is the second harmonic power; $D_1 = 1.43$, $\gamma = 5$, $\Delta = -3.2$.

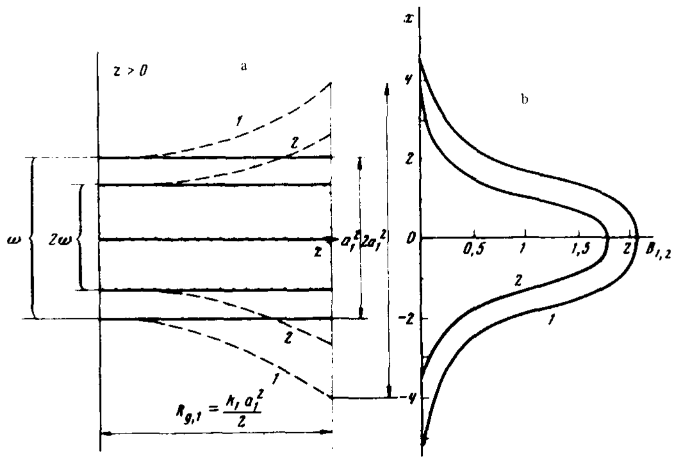

FIG. 3. Waveguide propagation of beams in a quadratic medium: a—schematic representation of coupled waveguides at frequencies ω and 2ω. The dashed lines show the diffraction divergence of Gaussian beams in a linear medium over the diffraction-spreading distance R_g; b—natural modes of stationary two-dimensional waveguide $[A_j(z,x) = B_j(x)\exp(-i\Gamma_j z)]$ at $\Delta = 0$.

$$z_\Delta = \gamma^{-1}\ln\frac{\gamma}{\Delta}. \qquad (5)$$

It is seen from the comparison that for the diffracting beams the role of Δ is assumed by the diffusion coefficient D.

The nonlinear interaction of bounded diffracting beams changes also the phase in the beam cross section. Of unusual interest is the fact that these changes are such that mutual focusing of the fundamental radiation and of the harmonic takes place both inside the nonlinear layer and behind it. This is clearly seen from the beam-intensity plots of Figs. 1 and 2. Figure 1 shows mutual focusing of beams after emerging from the crystal, while Fig. 2 shows mutual focusing for the focused fundamental-radiation beam. In a linear medium, the intensity of the fundamental beam at the focus $z = 0.5$ would exceed the initial value by 10 times, and in the case of harmonic generation the total intensity exceeds the initial value by 40 times. It is precisely this circumstance, in our opinion, which can explain the experimentally observed[3] breakdown of crystals in synchronous generation of optical harmonics.

4. The change in the phase fronts during the course of the interaction can lead not only to mutual focusing of the waves, but, if the boundary conditions are suitably chosen, also to waveguide propagation of two beams at frequencies ω and 2ω. It must be emphasized that we are dealing here with focusing and waveguide propagation in the medium with only quadratic nonlinearity. In this sense, the regime in question differs in principle from the known phenomenon of self-focusing and waveguide propagation in media with cubic nonlinearity.

We considered the case when two beams of frequency ω and 2ω, with comparable powers, are incident on a quadratic medium from the outside (see Fig. 3). If the phase asynchronism integral I_2 is negative, then it can be shown that the maximum of the amplitudes of the beams is bounded from below, $\max|A_j| \geq |I_2|/I_1$, so that the breakup of the field into two noninteracting beams becomes impossible, and the beams must propagate in the form of coupled waveguides, which oscillate in the general case. Sufficiently strongly focused or diverging beams incident on a quadratic medium have a positive integral I_2 and may not enter in the mutual-capture regime. For Gaussian beams with plane phase fronts and comparable powers, the condition of mutual capture means comparability of the diffraction-spreading length $R_d \sim D_1^{-1}$ with the nonlinear-interaction length $L_{nl} \sim \gamma^{-1}$ $(\gamma D_1^{-1} \lesssim 1)$. The phases of the beams at the entrance should then satisfy the relation $|2\phi_1 - \phi_2| < \pi/2$.

Stationary waveguide propagation corresponds to beams with constant amplitude profile

$$A_j = \gamma^{-1}\Gamma_1 B_j(\bar{x},\bar{y})e^{-i\Gamma_j z}, \quad \Gamma_2 = 2\Gamma_1 - \Delta. \qquad (6)$$

Substituting (6) in (1) and (2) we obtain equations for the natural modes:

$$\bar{\Delta}_\perp B_1 = B_1(1 - B_2), \qquad (7)$$

$$\bar{\Delta}_\perp B_2 = 4\left(1 - \frac{\Delta}{2\Gamma_1}\right)B_2 - 2B_1^2, \quad \bar{x} = \sqrt{\Gamma_1 D_1^{-1}}\,x, \quad \bar{y} = \sqrt{\Gamma_1 D_1^{-1}}\,y \qquad (8)$$

In the case of two-dimensional beams, the system (7)—(8) is a Hamiltonian system with an energy integral

$$H = 4(B_1')^2 + (B_2')^2 - 4\left[B_1^2 + \left(1 - \frac{\Delta}{2\Gamma_1}\right)B_2^2 - B_1^2 B_2\right]. \qquad (9)$$

Solutions bounded at infinity $(x \to \pm\infty)$ correspond to $H = 0$. The numerical solution of the system (7)—(8) at $\Delta = 0$ is shown in Fig. 3b. Here $P_1 = 4.8D_1^2\gamma^{-2}a^{-3}$, $P_2 = 2.76D_1^2\gamma^{-2}a^{-3}$. If $\Gamma_1 = 2\Delta/3$ and $\Gamma_2 = \Delta/3$, then there exists a degenerate mode with $B_1 = (3/2\sqrt{2})\cosh^{-2}(x/2)$ and $B_2 = \sqrt{2}B_1$. With this degeneracy, a mode of cylindrical beams was obtained numerically; here $P_1 = (9/8)D_1^{2}{}^{-2}a^{-3}$ and $P_1 = 2P_1$.

5. Thus, the critical power of the produced waveguides of the fundamental and second harmonics is, apart from a numerical factor,

$$P_{cr} \sim \frac{c\lambda^4}{\chi^2 a^2}. \qquad (10)$$

For typical crystals with $\chi \approx 10^{-8}$ cgs esu and for beams with $a = 0.1$ mm and $\lambda_1 = 0.5$ μ, we have relatively low powers $P_{cr} \approx 100$ W, which is readily attainable in experiments.

We note in conclusion that mutual-focusing effects become more strongly manifest in the case of nondegenerate three-frequency interactions.

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