Concerning one analytic property of the amplitudes of reactions in which three particles are produced in the final state

A. I. Baz' and S. P. Merkur'ev

Leningrad State University

(Submitted October 21, 1974)
ZhETF Pis. Red. **20**, No. 11, 753–754 (December 5, 1974)

We describe the features of the three-particle scattering amplitude (2→3) under the condition that the two-particle subsystems have resonant states.

We consider a system of three nonrelativistic spinless particles a_i $(i=1,2,3)$, interacting with one another via a potential $v_{ij}(|r_{ij}|)$ with a finite effective radius. Let the potentials be such that every (and possibly not every) pair of particles a_i, a_j $(ij\equiv\alpha)$ has besides a bound state also one resonant state (for simplicity, s-state) characterized by a complex energy $(\epsilon_{\alpha 0}-i\Gamma_\alpha/2)$ with $\epsilon_{\alpha 0}>0$. We present below the results of an investigation of the influence of these resonances on the analytic properties of the amplitude A_α of the reaction in which the collision between the bound state (a_i+a_j) of the pair of particles a_i and a_j with the third particle leads to complete disintegration

$$(a_i+a_j)+a_e\to a_1+a_2+a_3. \qquad (1)$$

We shall use the three pairs $\{k_\alpha, p_\alpha\}$ of Jacobi momenta, where, for example, k_{12} is the relative momentum of the pair a_1 and a_2, and p_{12} is the momentum of the particle a_3 (all in the c.m.s.). The corresponding pair of reduced masses will be designated m_{12} and μ_{12}, so that the kinetic energy is $E=k_{12}^2/2m_{12}+p_{12}^2/2\mu_{12}$.

The amplitude A_α is a function of the relative momenta of the particles in the final state, and it is possible to use any pair $\{k_\beta, p_\beta\}$, since all three pairs can be linearly expressed in terms of one another. In addition, of course, A_α depends on the relative momentum p_α' of the colliding particles: $A=A_\alpha(k_\beta, p_\beta|p_\alpha')$. The amplitude is normalized in such a way that the differential cross section of the reaction (1) is

$$\frac{d^3\sigma}{d\hat{k}_\beta\, d\hat{p}_\beta\, d\epsilon_\alpha}=\frac{\sqrt{\epsilon_\alpha(E-\epsilon_\alpha)}}{p_\alpha'}\, m_\alpha(2\pi)^4|A_\alpha(k_\beta;p_\beta|p_\alpha')|^2. \qquad (2)$$

Here $\hat{k}$ and $\hat{p}$ are unit vectors in the directions of k and p, ϵ_α is the relative energy of the pair α $(\epsilon_\alpha=k_\alpha^2/2m_\alpha)$, and E is the total energy.

The system in question can be investigated with the aid of the Faddeev equations. A rather elaborate proof, which will not be presented here, shows that if the system has no three-body resonance, then the two-particle resonances lead to the following general form of the amplitude:

$$A_\alpha(k_\beta, p_\beta|p_\alpha')=\sum_\gamma\left\{\frac{A_{\alpha\gamma}}{\epsilon_\gamma-\left(\epsilon_{\gamma 0}-i\frac{\Gamma_\gamma}{2}\right)}+\tilde{A}_{\alpha\gamma}\right\}. \qquad (3)$$

Here $A_{\alpha\gamma}$ and $\tilde{A}_{\alpha\gamma}$ are bounded and smoothly-varying functions of the momenta k_γ and p_γ. No new singularities apart from poles appear in the amplitude A_α. This is a rather unexpected result, since one might think that the aggregate of two-particle resonances can lead to the appearance of more complicated singularities in the amplitudes A_α than simple poles in the pair energies.

Since the cross section (2) is proportional to the amplitude square, the cross section contains not only resonant parts but also all possible crossing terms. As a result, the cross section can assume a rather intricate form, especially when the singularities can overlap. We note one circumstance that may be useful in the analysis of the experimental cross sections. The cross section (2) integrated with respect to the angles of $\hat{k}_\alpha$ and $\hat{p}_\beta$ contains only one resonant term of the type $[(\epsilon_\beta - \epsilon_{\beta 0})^2 + (\Gamma_\beta/2)^2]^{-1}$. All the remaining resonant terms are "interested." This can be verified directly by calculating the corresponding integrals with respect to the angles.